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بکارگیری پایه‌های گروبنر برای طراحی ماشین‌آلات عظیم

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چکیده: با توجه به کاربرد بسیار فراوان پایه‌های گروبنر در علوم مهندسی، در این تحقیق نحوه بکارگیری پایه‌های گروبنر در چندجمله‌ای‌های بزییر برای طراحی اجزاء ماشین‌آلات بزرگ ارائه می‌شود.

کلمات کلیدی: چندجمله‌ایها-مکعب بزییر-پایه‌های گروبنر-طراحی اجزاء

مقدمه

یکی از دلایل پارامتری کردن یک منحنی یا رویه رسم آسان آن به کمک کامپیوتر است. یک کاربرد از این روش در طراحی هندسی به کمک کامپیوتر است به عنوان مثال برای طراحی بدنه ماشین‌آلات یا بال‌های هواپیما مهندس طراح نیاز به منحنی‌ها و رویه‌هایی دارد که دارای گوناگونی رنگ در شکل آنهاست و این باعث شرح آسان و سرعت در طراحی می‌شود که معادلات پارامتری شامل چندجمله‌ای‌ها و توابع گویا برای این کار مناسب‌تر هستند. که در این مقاله قصد معرفی پایه های گروبنر و نحوه عملکرد آن در مهندسی مکانیک شاخه طراحی اجزاء را داریم.

مفاهیم اولیه

فرض کنید $k[x_1, \cdots, x_n]$ یک حلقه چندجمله‌ای با n متغیر روی میدان k باشد. فرض کنید $f_1, \cdots, f_s$ چندجمله‌ای‌هایی در $k[x_1, \cdots, x_n]$ باشد آنگاه نمایش $< f_1, \cdots, f_s >$ یک ایده‌ال بطور متناهی تولید شده بوسیله چندجمله‌ای‌های انتخاب شده $f_1, \cdots, f_s$ است. ما این چندجمله‌ای‌ها را فرم یک پایه از ایده‌ال گوییم. حال نماد $\mathbf{V}(f_1, \cdots, f_s)$ را واریته آفین تولید شده بوسیله $f_1, \cdots, f_s$ گوییم و آن زیرمجموعه‌ای ناتهی از k^n است که شامل همه صفرهای معمولی $f_1, \cdots, f_s$ می‌باشد یعنی:

$$\mathbf{V}(f_1, \cdots, f_s) = \{(a_1, \cdots, a_n) \in k^n \mid f_i(a_1, \cdots, a_n) = 0 \forall i, 1 \leq i \leq s\}$$

برای تعریف پایه‌های گروبنر در ایده‌ال‌های $k[x_1, \cdots, x_n]$ ، ابتدا به مفهوم یک رابطه ترتیبی نیازمندیم.

تعریف: یک ترتیب تک جمله‌ای روی $k[x_1, \cdots, x_n]$ ، یک رابطه $>$ روی $\mathbf{Z}^n_{\geq 0} = \{(\alpha_1, \cdots, \alpha_n) | \alpha_i \in \mathbf{Z}_{\geq 0}\}$ (بطور معادل هر رابطه روی یک مجموعه‌ای از تک‌جمله‌ای‌ها $x^\alpha, \alpha \in \mathbf{Z}^n_{\geq 0}$) است که در شرایط زیر صدق کند:

۱- $>$ یک ترتیب جمعی (خطی) روی $\mathbf{Z}^n_{\geq 0}$ باشد.

$$(\forall \alpha, \beta \in \mathbf{Z}^n_{\geq 0}, \alpha > \beta, \alpha < \beta, \alpha = \beta)$$

۲- اگر $\alpha < \beta$ و $\gamma \in \mathbf{Z}_{\geq 0}$ آنگاه $\alpha + \gamma < \beta + \gamma$.

۳- $>$ روی $\mathbf{Z}^n_{\geq 0}$ خوشترتیب باشد (هر زیرمجموعه ناتهی دارای عضو ابتدا باشد).
در اینصورت گوییم $x^\alpha > x^\beta$ وقتی $\alpha > \beta$.

تعریف: فرض کنید $f = \Sigma_\alpha a_\alpha x^\alpha$ یک چندجمله‌ای غیر صفر در $k[x_1, \cdots, x_n]$ باشد در اینصورت:

۱) بزرگترین درجه f را بفرم زیر تعریف می‌کنیم:

$$multideg(f) = max\{\alpha \in \mathbf{Z}^n_{\geq 0} : a_\alpha \neq 0\}$$

که ماکزیمم بر حسب ترتیب اتخاذ شده $>$ است.

۲) ضریب پیشرو چندجمله‌ای f را بصورت $LC(f) = a_{multideg}(f)$ نشان می‌دهیم.

۳) تک‌جمله‌ای پیشرو چندجمله‌ای f را بصورت $LM(f) = x^{multideg}(f)$ نشان می‌دهیم.

۴) جمله پیشرو چندجمله‌ای f را بصورت $LT(f) = LC(f).LM(f)$ نشان می‌دهیم.

تعریف (ترتیب lexicographic): فرض کنید $\alpha = (\alpha_1, \cdots, \alpha_n)$ و $\beta = (\beta_1, \cdots, \beta_n)$ توان‌های تک‌جمله‌ای‌ها باشند که بصورت بردار نمایش داده شده باشند و $|\alpha| = \Sigma\alpha_i$ و $|\beta| = \Sigma\beta_i$ مجموع درجه‌ها باشند، آنگاه $\alpha >_{lex} \beta$ اگر در تفاضل بردارهای $\alpha - \beta \in \mathbf{Z}^n$، چپ‌ترین درایه‌ی غیر صفر مثبت باشد.

معرفی پایه‌های گروبنر

تقسیم هر چندجمله‌ای f به لیستی از چندجمله‌ای‌های $f_1, \cdots, f_s$ نیازمند الگوریتم تقسیم تعمیم یافته است.

(**الگوریتم تقسیم**) یک ترتیب تک‌جمله‌ای را در نظر بگیرید. فرض کنید $F = (f_1, \cdots, f_s)$ یک s-تایی مرتب از چندجمله‌ای‌ها باشد آنگاه هر $f \in k[x_1, \cdots, x_n]$ را می توان بفرم زیر نوشت:

$$f = a_1 f_1 + \cdots + a_s f_s + r$$

که $a_i, r \in k[x_1, \cdots, x_n]$ و $r = 0$ یا r یک ترکیب خطی با ضرایب در k از تک‌جمله‌ای‌هایی است که قابل تقسیم بوسیله هیچ یک از $LT(f_1), \cdots, LT(f_s)$ نیست. r را باقیمانده f روی تقسیم با F گوییم و اگر $a_i f_i \neq 0$ آنگاه داریم:

$$multideg(f) \geq multideg(a_i f_i)$$

توجه داشته باشید باقیمانده r که در بالا ذکر شد منحصر به فرد نیست و به ترتیب قرار گرفتن چندجمله‌ای‌ها در F و ترتیب تک‌جمله‌ای که اتخاذ می‌شود، وابسته است. این مشکل زمانی از بین می‌رود که تقسیم چندجمله‌ای‌ها بر یک پایه‌های گروبنر انجام پذیرد. ایده‌ال $I \subset k[x_1, \cdots, x_n]$ را یک ایده‌ال تک‌جمله‌ای گوییم هرگاه توسط یک مجموعه از تک‌جمله‌ای‌ها تولید شود یعنی یک زیرمجموعه $A \subset \mathbf{Z}^n_{\geq 0}$ وجود داشته باشد، بطوریکه I شامل همه چندجمله‌ای‌هایی باشد که بصورت مجموع متناهی به شکل $\sum_{\alpha \in A} h_\alpha x^\alpha$ که در آن $h_\alpha \in k[x_1, \cdots, x_n]$. در این مورد می‌نویسیم $I =< x^\alpha \mid \alpha \in A >$.

تعریف: فرض کنید $I \subset k[x_1, \cdots, x_n]$ یک ایده‌ال غیر صفر باشد آنگاه $LT(I)$ ،مجموعه‌ای از جملات پیشرو اعضای I است و $< LT(I) >$ ایده‌ال تولید شده توسط اعضای $LT(I)$است.

لم دیکسون: فرض کنید $I =< x^\alpha | \alpha \in A >\subset k[x_1, \cdots, x_n]$ یک ایده‌ال تک‌جمله‌ای باشد در اینصورت $\alpha(1), \cdots, \alpha(s) \in A$ چنان موجودند که $I =< x^{\alpha(1)}, ..., x^{\alpha(n)} >$. بعبارت دیگر I یک مولد متناهی دارد.

قضیه پایه هیلبرت: هرایده‌ال I از $k[x_1, \cdots, x_n]$ یک مجموعه مولد متناهی دارد یعنی $g_1, \cdots, g_t \in I$چنان موجودند که $I =< g_1, \cdots, g_t >$.

قضیه (وضعیت زنجیر صعودی ACC):فرض کنید $I_1 \subset I_2 \subset \cdots$ یک زنجیر صعودی از ایده‌ال های در $k[x_1, \cdots, x_n]$ باشند آنگاه یک $N \geq 1$ وجود دارد

بطوریکه $I_N = I_{N+1} = I_{N+2} = \cdots$.

تعریف: یک ترتیب تک‌جمله‌ای را در نظر بگیرید. یک زیر مجموعه متناهی $G = \{g_1, \cdots, g_t\}$ از یک ایده‌ال I را یک پایه‌های گروبنر گوییم هرگاه

$$< LT(I) > = < LT(g_1), \cdots, LT(g_t) > .$$

هرگاه تقسیم f بر پایه‌های گروبنر G انجام شود، باقیمانده r یکتا شده و آن را به صورت $r = \overline{f}^G$ نشان می‌دهیم. پایه‌های گروبنر با الگوریتم‌های گوناگونی محاسبه می‌شود که اولین آنها که معروف‌ترین‌شان نیز هست الگوریتم بوخبرگر نام دارد. این الگوریتم با استفاده از S−چندجمله‌ای‌ها به محاسبه پایه‌های گروبنر می‌پردازد.

تعریف: S−چندجمله‌ای، دو چندجمله‌ای $f_1, f_2 \in k[x_1, \cdots, x_n]$ را به صورت زیر تعریف می‌کنیم:

$$S(f_1, f_2) = \frac{x^\gamma}{LT(f_1)}.f_1 - \frac{x^\gamma}{LT(f_2)}.f_2$$

که $x^\gamma = LCM(LM(f_1), LM(f_2))$ و $LM(f_i)$ وابسته به ترتیبی است که اتخاذ شده است.

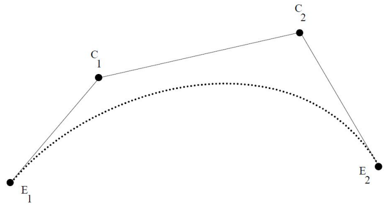
قضیه بوخبرگر:یک پایه $\{g_1, \cdots, g_s\} \in I$ یک پایه‌های گروبنر I است اگر و تنها اگر

$\overline{S(g_i, g_j)}^G = 0$ برای $1 \leq i < j \leq s$

مکعب بزییر

در این قسمت به معرفی مختصر چندجمله‌ای بزییر و کاربرد پایه‌های گروبنر در چندجمله‌ای درجه سوم بزییر (مکعب بزییر) می‌پردازیم. این منحنی باید دارای ویژگی‌هایی باشد مثلاً شیب خط مماس خروجی از E_1 باید با شیب خط واصل E_1 و C_1 یکسان باشد و به همین دلیل C_1 را نقطه کنترل می‌گویند که موقعیت C_1 با E_1 تعیین کننده شیب منحنی است که از نقطه E_1 شروع شده و به E_2 ختم می‌شود. در حالت کلی نیاز نیست منحنی از نقاط و یا نزدیکی C_1 و C_2 بگذرد. همین شرایط برای E_2و C_2 نیز برقرار است.

بنابراین C_1 و C_2 مستقل از هم بوده و وابسته به نقاط انتهایی‌شان هستند. برای درک بهتر موضوع به شکل زیر توجه کنید.



بنابراین می‌توانم با روش‌های بالا منحنی‌های زیادی رسم کنیم اما این کار باید با روش‌های ریاضی همراه باشد. معادله چندجمله‌ای درجه سوم (مکعب) Bezier بطور پارامتری بصورت زیر است:

$$X(t) = (1-t)^3x_1 + 3t(1-t)^2x_2 + 3t^2(1-t)x_3 + t^3x_4$$
$$Y(t) = (1-t)^3y_1 + 3t(1-t)^2y_2 + 3t^2(1-t)y_3 + t^3y_4$$

که (x_i, y_i) برای $i = 1, 2, 3, 4$ مختصات ۴ نقطه کنترل و پایانی هستند و $0 \leq t \leq 1$ می‌باشند.

کاربرد پایه‌های گروبنر در مکعب بزییر برای طراحی اجزاء

در یکی از کاربردهای پایه‌های گروبنر برای مکعب بزییر که در این مقاله نشان خواهیم داد، چطور می‌توان پارامتر t را حذف کرد و یک چندجمله‌ای یافت که معرف مکعب باشد. این کار را با محاسبه یک پایه‌های گروبنر کاهش یافته برای ایدال I متشکل از معادلات پارامتری مکعب بزییر و متغیر x و y بصورت $I =< x - X, y - Y >$ با ترتیب حذفی $lexdeg([t], [x, y, x_i, y_i])$ انجام می‌دهیم. این پایه‌های گروبنر شامل ۱۲ چندجمله‌ای است که فقط یکی از آنها پارامتر t را ندارد و آن $g \in R[x, y, x_i, y_i]$ است و g همگن از درجه ۶ و شامل ۴۶۰ جمله می‌باشد.

یک مثال به عنوان نمونه، با نقاط کنترل بترتیب $(\frac{3}{2},0)$ و $(0,\frac{1}{4})$ و $(3,2)$ و $(2,0)$ بررسی می‌کنیم و با جایگذاری نقاط کنترل در معادلات پارامتری و تشکیل ایدهال I و محاسبه پایه‌های گروبنر برای I با ترتیب ذکر شده چندجمله‌ای g بصورت زیر بدست می‌آید:

$$g=-213948x+66420y-214164yx-145656y^2x+89964x^2y+110079x^2+135756+78608y^3-18522x^3+219456y^2=0$$

نتایج

روش‌های ذکر شده یک سری مزایا و معایب دارند که بصورت زیر برمی‌شماریم:

مزایا:

۱- می‌توان منحنی‌ها را راحت‌تر رسم کرد.

۲- یافتن گره‌های ابتدایی و انتهایی برای المان محدود برای هر دقت مطلوب.

معایب:

پیچیدگی محاسبات پایه‌های گروبنر و نیز چندجمله‌ای g برای مکعب بزییر.

الگوریتم اجراء شده مثال بالا در نرم افزار:

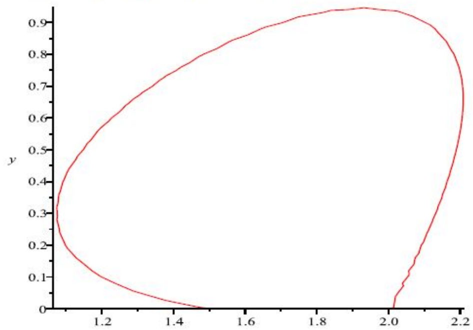
```
> ########## polynomial consist just x,y ##########
> single:=proc(F,ord)
> local i,g,A:
> A:=Basis(F,ord):
> g:=[]:
> for i from 1 to nops(A) do
> if indets(A[i])={x,y} then
> g:=A[i]:
> print('g'=-1*g);
> fi:
> od:
> end:
> single(F,lexdeg([t],[x,y]));
```

$g=135756+110079*x^2-214164yx+219456y^2-213948x+66420y-145656y^2x+78608y^3+89964x^2y-18522x^3$

```
> with(plots):
> g:=-213948*x+66420*y-214164*y*x-145656*y^2*x+89964*x^2*y
> +110079*x^2+135756+78608*y^3-18522*x^3+219456*y^2;
```

$g:=135756+110079*x^2-214164yx+219456y^2-213948x+66420y-145656y^2x+78608y^3+89964x^2y-18522x^3$

```
> implicitplot(g,x=0..2.3,y=0..1);
```



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Algebraic Structure of Concentrators of a Measure

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Abstract: In this paper we show that for a positive measure λ, the family of all measurable set that λ is concentrated on them, is a lattice but is not complete in general. Also this family is an up-set of family of all measurable sets but it is not principal up-set in general.

Keywords: Lattice, Complete Lattice, Up-set, Concentrator of measure.

1 INTRODUCTION

Always communication between two science or two branches of a science is very useful and can be improve both of branches. One of the most applicable theories of mathematics is a measure theory and also the concept of concentration of a measure on a set is a fundamental concept in measure theory. This concept is necessary for the Lebesgue-Radon-Nikodiom theorem. On the other hand the notion of lattice, plays an important role in algebra and even in other sciences. In this paper we show that for a measure space by a positive measure, the family of measurable sets which measure of measure space is concentrated on them is a lattice but it is not complete lattice in general and also we show that it is an up-set of family of all measurable sets but it is not principal.

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2 Preliminaries

In this section we recall some definitions from ordered algebra and measure theory:

Definition 2.1. [2]. *Let $(E, \leq)$ be an ordered set. Let $x, y \in E$. We say that x, y are comparable if either $x \leq y$ or $y \leq x$. In contrast, we say that x, y are incomparable, and write $x \parallel y$, when xy and yx.*

Definition 2.2. [2]. *Let $(E, \leq)$ be an ordered set. A subset U of E is called an up-set if $x \in U$ and $y \in E$ such that $x \leq y$, then $y \in U$. Also a principal up-set is an up-set of the form $x^{\uparrow} = \{y \in E \mid x \leq y\}$.*

Definition 2.3. [2]. *A lattice is an ordered set in which every pair of elements (and hence every finite subset) has an infimum and a supremum. Infimum and supremum of x, y is denoted by $x \wedge y$ and $x \vee y$ respectively. Thus a lattice often denote* *by* $(E, \wedge, \vee, \leq)$.

Definition 2.4. [2]. *A lattice* L *is said to be complete if every subset of* L *has an infimum and a supremum.*

Definition 2.5. [2]. *A sublattice of a lattice* L *is a nonempty subset* E *of* L *such that if* $x, y \in E$, *then* $x \wedge y \in E$ *and* $x \vee y \in E$.

Definition 2.6. [2]. *A filter of a lattice* L *is a sublattice of* L *that is also up-set.*

Definition 2.7. [2]. *A filter* F *of lattice* L *is called prime filter if* $F \neq L$ *and if* $a \vee b \in F$, *then* $a \in F$ *or* $b \in F$.

Definition 2.8. [4]. *Let* λ *be a measure on a* σ*-algebra* M. *If there is a set* $A \in M$ *such that* $\lambda(E) = \lambda(A \cap E)$ *for every* $E \in M$, *we say that* λ *is concentrated on* A. *This is equivalent to the hypothesis that* $\lambda(E) = 0$ *whenever* $A \cap E = \phi$.

Definition 2.9. [4] *Let* μ *be a positive measure on a* σ*-algebra* M *and* λ *be an arbitrary measure on* M. *We say that* λ *is absolutely continuous with respect to* μ *and write* $\lambda \ll \mu$, *if* $\lambda(E) = 0$ *for every* $E \in M$ *for which* $\mu(E) = 0$.

Definition 2.10. [4]. *Suppose* λ *and* μ *are measures on* M, *and suppose there exist a pair of disjoint sets* A *and* B *such that* λ *is concentrated on* A *and* μ *is concentrated on* B. *Then we say that* λ *and* μ *are mutually singular, and write* $\lambda \perp \mu$.

3 Main Results

Let (X, λ, M) be a measure space. We consider C_λ as family of all measurable set which λ is concentrated on them. ie;

$C_\lambda = \{A \in M \mid \lambda \text{ is concentrated on } A\}$.

Theorem 3.1 $C_\lambda = M$ if and only if $\lambda = 0$.

Theorem 3.2 If $A, B \in C_\lambda$, then $A \cap B \in C_\lambda$.

Theorem 3.3 Let λ be a positive measure. If $A \in C_\lambda$ and $A \subseteq B$, then $B \in C_\lambda$.

By *Definition* 2.8 if (X, λ, M) be a measure space and $A \in C_\lambda$, then clearly $\lambda(A) = \lambda(X)$. Now see the next theorem.

Theorem 3.4 Let (X, λ, M) be a measure space and λ be a finite positive measure. If $A \in M$ and $\lambda(A) = \lambda(X)$, then $A \in C_\lambda$.
Proof. Let $E \in M$ and $A \cap E = \phi$. Let $D = A \cup E$. Thus $\lambda(D) = \lambda(E) + \lambda(A)$ and so $\lambda(D)\lambda(A)$. On the other hand $\lambda(D)\lambda(X) = \lambda(A)$. Hence $\lambda(E) = 0$ and so $A \in C_\lambda$.

Next example shows that finiteness condition in the previous theorem, is necessary:

Example 3.5 Let X be the set of all real numbers, $A = [0, +\infty)$ and λ be Lebesgue measure. Then $\lambda(A) = \lambda(X)$ but $A \notin C_\lambda$ because $3 = \lambda([-1, 2]) \neq \lambda([-1, 2] \cap A) = 2$.

Theorem 3.6 If λ is a positive measure, then $(C_\lambda, \subseteq)$ is a lattice but it is not necessary complete lattice.

Proof. Suppose that $A, B \in C_\lambda$. By Theorem 3.2, $A \cap B \in C_\lambda$ and since $A \subseteq A \cup B$ by Theorem 3.3, $A \cup B \in C_\lambda$. Thus $Sup\{A, B\} = A \cup B$ and $Inf\{A, B\} = A \cap B$ are in C_λ. Hence $(C_\lambda, \subseteq)$ is a lattice.
Now let $X = [0, 1]$ and λ be the Lebesgue measure. For each $x \in X$ we put $A_x = X - \{x\}$. Then $\lambda(A_x) = \lambda(X) = 1$ and by Theorem 3.4, $A_x \in C_\lambda$ for each $x \in X$. But $\bigcap_{x \in X} A_x = \phi$ and is not in C_λ. Therefore C_λ is not complete.

Theorem 3.7 Let λ be a positive measure and $A \in C_\lambda$ and $B \subseteq A$. If $\lambda(B) = 0$, then $A - B \in C_\lambda$.

Theorem 3.8 Let (X, λ, M) be a measure space and λ be a positive measure. Then C_λ is an up-set of an ordered set $(M, \subseteq)$ but it is not principal in general.

Proof. Theorem 3.3 shows that C_λ is an up-set of M. Now we show that it is not necessary principal. Let X be the set of all real numbers and λ be the Lebesgue measure. Let $A \in M$ such that $C_\lambda = A^\uparrow$. Thus $A \in C_\lambda$ and so $\lambda(A) = \lambda(X)$. Hence A is uncountable. Suppose B is a nonempty countable subset of A. Then $\lambda(B) = 0$ and by Theorem 3.7, $A - B \in C_\lambda$. This means that $A \subseteq A - B$ and this is a contradiction. Thus C_λ is not principal up-set.

Theorem 3.9 Let (X, λ, M) be a measure space and λ be a positive measure. Then C_λ is a filter of $(M, \subseteq)$ but it is not necessary prime.

Let (X, M) be a measurable space and $C(X, M) = \{C_\lambda \mid \lambda \ is \ a \ measure \ on \ M\}$. Thus $(C(X, M), \subseteq)$ is an ordered set and we have the following theorems:

Theorem 3.10 Let (X, M) be a measurable space and λ and μ be tow non zero measures on M. If $\lambda \perp \mu$ then $C_\lambda \parallel C_\mu$.

Theorem 3.11 Let (X, M) be a measurable space and λ and μ be tow measures on M. If $\lambda \leq \mu$ then $C_\mu \subseteq C_\lambda$.

Theorem 3.12 Let (X, M) be a measurable space and λ and μ be tow measures on M. If $\lambda \ll \mu$ then $C_\mu \subseteq C_\lambda$.

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Groups with claw-free non-commuting graphs

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Abstract: Given a non-abelian group G, the non-commuting graph of G is the graph with the vertex set $G - Z(G)$, where two non-central elements x and y are joined by an edge if and only if $xy \neq yx$, where $Z(G)$ is the center of G. In this paper, we prove that every finite group G with claw-free non-commuting graph is isomorphic to one of the groups S_3, D_8 or Q_8.
Keywords: non-commuting graphs, claw-free graphs.

1 INTRODUCTION

For an integer $z > 1$, we denote by $\pi(z)$ the set of all prime divisors of z. If G is a finite group, then $\pi(|G|)$ is denoted by $\pi(G)$. Let G be a non-abelian group and $Z(G)$ be its center. We will associate a graph $\Gamma(G)$ to G which is called the non-commuting graph of G. The vertex set $V(\Gamma(G))$ is $G - Z(G)$ and the edge set $E(\Gamma(G))$ consists of (x, y), where x and y are distinct non-central elements of G such that $xy \neq yx$. The commuting graph associated to a non-abelian group G is the complement of $\Gamma(G)$. Here, we are considering simple graphs, i.e., graphs with no loops or directed or repeated edges. The non-commuting graphs of the non-abelian finite groups have been studied in some literatures. In [1], some properties of non-commuting graph have been studied. A set of vertices of a graph Γ is called an independent set, if its elements are pairwise nonadjacent. Throughout this paper, let G be a finite group and $M(G)$ denote a set of the orders of maximal abelian subgroups G. The independent number of a graph Γ, which is denoted by $\alpha(\Gamma)$, is the cardinality of the largest its independent set. Also, a claw-free graph is a graph that does not have a claw, the complete bipartite graph $K_{1,3}$ as an induced subgraph. Because of the special properties of claw-free graphs, they have been studied in some papers. In this paper, we are going to study the groups that their non-commuting graphs are claw-free.

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2 Some Lemmas

In this section, we bring some lemmas which will be used in the proof of the main theorem:

Lemma 2.1. *If $M(G) \subseteq \{2, 3\}$, then $G \cong S_3$.*

Proof. Since for every $x \in G$, $\langle x \rangle$ is an abelian subgroup of G, we deduce that G is a finite group and $\pi(G) \subseteq \{2, 3\}$. Also, for every $p \in \pi(G)$, the subgroups of order p^2 are abelian, so $|G|_2 = 2$ and $|G|_3 = 3$. But G is non-abelian and hence, $\pi(G) = \{2, 3\}$. Thus lemma follows. □

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Lemma 2.2. *If $M(G) \subseteq \{2, 4\}$, then $G \cong D_8$ or Q_8.*

Proof. Since G is non-abelian, $M(G) \subseteq \{2, 4\}$ and for every $x \in G$, $\langle x \rangle$ is an abelian subgroup of G, G is a 2-group, $|Z(G)| = 2$ and there exists $x \in G$ such that $O(x) = 4$. Also, we can see at once that $G/Z(G)$ is a 2-elementary abelian group and $C_G(x) = \langle x \rangle$. Thus $\langle x \rangle/Z(G)$ is a normal subgroup of $G/Z(G)$ and hence, $\langle x \rangle$ is normal in G. Therefore, $G/\langle x \rangle = G/C_G(x) \hookrightarrow \mathrm{Aut}(\langle x \rangle) \cong Z_2$. This forces $|G| = 8$ and hence, lemma follows. □

Lemma 2.3. *If G is a finite group with $\alpha(\Gamma(G)) \leq 2$, then $G \cong S_3$, D_8 or Q_8.*

Proof. By [2], we can see that M is a maximal abelian subgroup of G if and only if $M - Z(G)$ is an independent set of $\Gamma(G)$. Thus "$\alpha(\Gamma(G)) \leq 2$" implies that for every maximal abelian subgroup M of G, $|M| - |Z(G)| \leq 2$. Thus $|Z(G)|(|M|/|Z(G)| - 1) \in \{1, 2\}$. This forces $(|M|, |Z(G)|) \in \{(2, 1), (3, 1), (4, 2)\}$. Thus one of the following holds:

(i) $|Z(G)| = 1$ and $M(G) \subseteq \{2, 3\}$. Then Lemma 2.1 forces $G \cong S_3$

(ii) $|Z(G)| = 2$ and $M(G) \subseteq \{2, 4\}$. Then Lemma 2.2 forces $G \cong D_8$ or Q_8 and hence, lemma follows.

□

Lemma 2.4. *If G is a finite group and H, K, L are distinct proper subgroups of G such that $G = H \cup K \cup L$, then $[G : H] = [G : K] = [G : L] = 2$ and $H \cap L = H \cap K = K \cap L = H \cap K \cap L$.*

Proof. It follows immediately by considering the order of G. □

2.1 Main results

Theorem 2.5. *If G is a non-abelian finite group such such that $\Gamma(G)$ is claw-free, then $G \cong S_3$, D_8 or Q_8.*

Proof. Let $M = \{x_1, ..., x_t\}$ be a maximal independent set of $\Gamma(G)$ such that $|M| = \alpha(\Gamma(G))$. We continue the proof in the following cases:

(i) Let $\alpha(\Gamma(G)) \geq 3$ and $x_{i_1}, x_{i_2}, x_{i_3}$ be three arbitrary elements of M. Since $\Gamma(G)$ is claw-free, we deduce that for every $y \in G - Z(G)$, $y \in C_G(x_{i_1})$, $C_G(x_{i_2})$ or $C_G(x_{i_3})$. This shows that $G = C_G(x_{i_1}) \cup C_G(x_{i_2}) \cup C_G(x_{i_2})$. Thus Lemma 2.4 shows that

$$\cap_{j=1}^{3} C_G(x_{i_j}) = C_G(x_{i_1}) \cap C_G(x_{i_2}). \quad (1)$$

Now let $z \in G - (M \cup Z(G))$. If there exists $1 \leq i, j \leq t$ such that $z \not\sim x_i$ and $z \not\sim x_j$, then by (1), for every $u \in \{1, .., t\} - \{i, j\}$, $z \not\sim x_u$. Thus $M \cup \{z\}$ is an independent set, which is a contradiction. Thus there exists at least one $i \in \{1, ..., t\}$ such that $z \not\sim x_i$ and hence, since $\Gamma(G)$ is claw-free and $\alpha(\Gamma(G)) \geq 3$, $\alpha(\Gamma(G)) = 3$. Thus [2] shows that there exists a maximal abelian subgroup M' of G such that $|M' - Z(G)| = 3$ and for every maximal abelian subgroup M'' of G, we have $|M'' - Z(G)| \leq 3$. Thus $|Z(G)| = 1$ and $3 \in M(G) \subseteq \{2, 3\}$. So Lemma 2.1 shows that $G \cong S_3$ and hence, $\alpha(\Gamma(G)) = 2$, which is a contradiction.

(ii) If $\alpha(\Gamma(G)) \leq 2$, then Lemmas 2.1 and 2.2 completes the proof.

□

Remark 2.6. *If G is a finite non-abelian group, then there exists $x, y \in G - Z(G)$ such that $xy \neq yx$. If $\Gamma(G)$ is triangular free, then we can see at once that $G = C_G(x) \cup C_G(y)$, which is impossible. Thus for every finite non-abelian group, $\Gamma(G)$ contains a triangular.*

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Characterization of some simple groups by the set of orders of maximal abelian subgroups

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Abstract: In this paper, we study the structure of finite groups which their sets of the order of maximal abelian subgroups are same with the set of the order of maximal abelian subgroups of $C_n(4)$, where $n \geq 17$ is odd.
Keywords: Maximal abelian subgroup, prime graph, maximal independent set.

1 INTRODUCTION

For a finite group G, put $M(G) = \{|H| : H$ is a maximal abelian subgroup of $G\}$. By $a(G)$ and $a_r(G)$, we denote the maximum number in $M(G)$ and the order of the largest abelian subgroup of r-Sylow subgroup of G, respectively. A finite group G is called *characterizable by the set of orders of maximal abelian subgroups*, if each finite group H with $M(G) = M(H)$ is isomorphic to G. For instance, alternating group A_n, where n and $n-2$ are primes or $n \leq 10$ and the simple group $B_n(q)$, where n is a power of 2, are characterizable by the set of orders of their maximal abelian subgroups (see [2] and [1]). In this paper, we prove that:

Main Theorem. Let G be a finite group and let $n \geq 17$ be an odd number. If $M(G) = M(C_n(4))$, then $G \cong\ C_n(4)$.

Throughout this paper, p is a prime number, $q = p^k$ and G is a finite group. We denote by $\pi(G)$ the set of prime divisors of the order of G. The *prime graph* $GK(G)$ of G is the graph with vertex set $\pi(G)$, where two distinct primes r and s are joined by an edge (we write $(r, s) \in GK(G)$) if G contains an element of order rs. An independent set in a graph Γ is a set of pairwise non-adjacent vertices. $\rho(G)$ ($\rho(r, G)$) denotes the independent set in $GK(G)$ (containing a prime r) with a maximal number of vertices. Let $t(G) = |\rho(G)|$ and $t(r, G) = |\rho(r, G)|$. We use $a \mid n$ when a is a divisor of n and use $|n|_a = a^e$, when $a^e \mid\mid n$, i.e., $a^e \mid n$ but $a^{e+1}n$. If a is a natural number, r is an odd prime number and $\gcd(r, a) = 1$, then by $e(r, a)$ we denote the minimal natural number n with $a^n \equiv 1 \pmod r$. The prime r with $e(r, a) = m$ is called a *primitive prime divisor* of $a^m - 1$. We denote by $r_m(a)$ some primitive prime divisor of $a^m - 1$. If a is odd, then let $e(2, a) = 1$ if $a \equiv 1 \pmod 4$ and let $e(2, a) = 2$ if $a \equiv -1 \pmod 4$. By Fermat's little theorem, $e(r, a) \mid r - 1$. Also, if $a^n \equiv 1 \pmod r$, then $e(r, a) \mid n$.

2 Main results

In the following, we have brought some useful lemmas which will be used during the proof of the main theorem:

Lemma 2.1. [2] *Let G and K be two finite groups such that $M(G) = M(K)$. Then the prime graph of G and the prime graph of K are same.*

Lemma 2.2. [4, Theorem 1] *Let G be a finite group with $t(G) \geq 3$ and $t(2, G) \geq 2$. Then the following hold:*

1. *There exists a finite non-abelian simple group S such that $S \leq \bar{G} = G/K \leq Aut(S)$ for the maximal normal soluble subgroup K of G.*
2. *For every independent subset ρ of $\pi(G)$ with $|\rho| \geq 3$ at most one prime in ρ divides the product $|K|.|\bar{G}/S|$. In particular, $t(S) \geq t(G) - 1$.*
3. *One of the following holds:*
   (a) *every prime $r \in \pi(G)$ non-adjacent to 2 in $GK(G)$ does not divide the product $|K|.|\bar{G}/S|$; in particular, $t(2, S) \geq t(2, G)$;*
   (b) *there exists a prime $r \in \pi(K)$ non-adjacent to 2 in $GK(G)$; in which case $t(G) = 3$, $t(2, G) = 2$, and $S \cong A_7$ or $A_1(q)$ for some odd q.*

For a finite integer n, we define the following function which will be used in some following lemmas:

$$\eta(n) = \begin{cases} n & \text{if } n \text{ is odd;} \\ \frac{n}{2} & \text{otherwise.} \end{cases}$$

Lemma 2.3. [5] *Let G be one of the simple groups of Lie type, $B_n(q)$ or $C_n(q)$, over a field of characteristic p. Let r, s be odd primes with $r, s \in \pi(G) \setminus \{p\}$. Put $k = e(r, q)$ and $l = e(s, q)$, and suppose that $1 \leq \eta(k) \leq \eta(l)$. Then r and s are nonadjacent if and only if $\eta(k) + \eta(l) > n$ and l/k is not an odd natural number.*

Lemma 2.4. [5, Proposition 3.1] *Let $G = C_n(q)$, $r \in \pi(G)$ and $r \neq p$. Then $(r, p) \notin GK(G)$ if and only if $\eta(e(r, q)) > n - 1$.*

Proof of the main theorem. If G is a finite group such that $M(G) = M(C_n(4))$, then Lemma 2.1 shows that $GK(G) = GK(C_n(4))$. Thus Lemma 2.4 guarantees that $\{2, r_{2n}\} \subset \rho(2, C_n(4)) = \rho(2, G)$. Since $t(G) \geq 3$ and $t(2, G) = t(2, C_n(4)) = 3$, Lemma 2.2(1) forces to exist the maximal solvable subgroup K of G and a finite non-abelian simple group S such that $S \leq \overline{G} = G/K \leq$ Aut(S) and $t(S) \geq t(G) - 1$. In [3], the authors show that $S \cong C_n(4)$. Now by considering the independent set

$$\rho = \{r_{2n}(4), r_{2(n-1)}(4), r_{2(n-2)}(4)\}$$

of $GK(G)$ and Lemma 2.2(2), we can see that $K = 1$. This implies that

$$C_n(4) \leq G \leq \text{Aut}(C_n(4)).$$

But

$$[\text{Aut}(C_n(4)) : C_n(4)] = 2.$$

Thus either $G \cong \text{Aut}(C_n(4))$ or $G \cong C_n(4)$. If $G \cong \text{Aut}(C_n(4))$, then G contains a field automorphism ψ of order 2. Thus $C_{C_n(4)}(\psi) = C_n(2)$. But $C_n(2)$ contains a maximal abelian subgroup M_0 of order $2^n + 1$. Thus G contains a maximal abelian subgroup M such that $M_0 \leq M$ and $[M : M_0] \mid 2$. But we can see at once that if there exists $a \in M(G)$ such that $r_{2n}(4) \mid a$, then $a = 4^n + 1$, which is a contradiction and hence, $G \cong C_n(4)$, as claimed.

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Spectral Spaces on G-Type Domains

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Abstract: This article introduces a new version of G-type domains that's called "pullback of G-type Domain" and for each G-type domain R, there exists a canonical associated pullback of G-type domain, that is denoted by "$\widetilde{R}$". Furthermore there exists a canonical induced map $Spec(R) \longrightarrow Spec(\widetilde{R})$ which is a homeomorphism. In special case, if R is a Prüfer G-type domain, then R will be coincides to $\widetilde{R}$.
As usual, if R is a commutative ring with unit, then "Spec(R)" will be equipped to Zariski topology, in addition for each topological space that is homeomorphic to the "$Spec(R) = X$" it is called a "Spectral Space".
Here, a new version of "G-type Space" is produced, which is a topological space homeomorphic to "$Spec(R) = X$", for a suitable G-type domain of R. The process of these spaces will be started by producing some of new definitions related to the concept of various version of Spectral Spaces .
Keywords: .

1 Some mathematical remarks

A domain R is called a "**G-type domain**" if its quotient field is countably generated as a R-algebra.
R is a G-type domain if and only if its zero ideal is the contraction of a maximal ideal in $R[x_1, x_2, ..., x_n, ...]$.
A prime ideal I of $R[x_1, x_2, ..., x_n, ...]$ is called "**G-type ideal**" if and only if its contraction in R and $R[x_1, x_2, ..., x_n]$ for all $n \geq 1$ are G-type.[1]
Let R be a domain with quotient field K and P be any prime ideal of R and $S = R \backslash P$ be a mcs set of R and $\bar{R} = R/P$ and T be the Total quotient ring of $\bar{R}$ so:
i) $\widetilde{R}$ is a pullback of a ring of fraction T of R such that each nonzero prime of T is contained in the union of height 1 primes.
ii) R^+ : the seminormalization of R.
iii) $R^{'}$: the integral closure of R.
iv) R^* : the complete integral closure of R.
v) Let $P(R) = \bigcap_{\substack{P \in spec(R) \\ P \neq (0)}} P$, it's shown that for brevity by P, so it's defined as following:

1) P^+: seminormalization of P.

2) $P^{'}$: integral closure of P.

3) P^*: complete integral closure of P.

vi) $X(R)$: denote the set of all valuation overrings of R.

$X^1(R)$: The set of all one-dimensional valuation overrings of R.
vii) m_V: denote the maximal ideal of any given valuation ring V.[7] The G-type domain R is called essential (essential type) if each nonzero prime ideal of R is contained in union of prime ideals in R with

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height 1.
Let the G-type domain of R is one dimensional and it's motivation $S^{-1}R$ is also essential, then the $\widetilde{R}$ is pullback of an essential G-type domain.[5] For each commutative ring R, let $Spec^i(A)$ denote the subspace of $Spec(R)$ consisting of the height i primes.
Especially, if $Spec^1(R) = \{P_1, P_2, ..., P_n\}$, then: $S^{-1}R = \cap R_{P_i}$.[5] Let P be a prime ideal in a ring R, then the following are equivalent:
i) P is a G-type ideal in R.
ii) There exists a countable multiplicative closed set S in R such that: P is maximal with respect to having the empty intersection with S.
iii) There are either only a countable number of prime ideals in R/P or any uncountable set of prime ideals properly containing P, say F, can be written in the form $F = \cup_{n\in\Lambda} F_n$, where Λ is a subset of the natural numbers, P is properly contained in $\cup_{Q\in F_n} Q$ for each n and some of the F_n are uncountable.[2] Let R be a domain, such that each of its ideals countably generated , then R is a G-type domain if and only if there exists a countably generated R-algebra "T" contains the quotient field of R.[14] Let K be an algebraically closed field and $R = K[x_1, x_2, ...]$ then every maximal ideal M of R is the form of $M = (x_1 - \alpha_1, x_2 - \alpha_2, ...)$ if and only if K is uncountable.[14] Let R has countable noetherian dimension, then R is a finite direct sum of G-type domain if and only if each localization R_P is a G-type domain or countably generated as a $\phi_P(R)$-algebra, where $\phi_P : R \longrightarrow R_P$ is the natural homomorphism.[14] The ring of R is said has the "CPA" property(Countable Prime Avoidance) if $A \subseteq \cup_{i=1}^{\infty} P_i$ (A an ideal of R) then $A \subseteq P_i$, $\exists i$.[14]

Let R be an integrally closed G-type domain, then:

i) $R^* = \cap\{V|V \in X^1(R)\}$

ii) $P(R) = P(R^*) = \cap\{m_V|V \in X^1(R)\}$

Let a ring R have "dcc" on finite intersections of prime ideals (R has dcc on prime ideals, R/P has only a countable number of nonzero minimal primes for each prime P), then each prime ideal P of R is a G-ideal (G-type ideal).

Let R be a noetherian domain, R is a G-type domain if and only if it has only countable number of nonzero minimal prime ideals

Let $k - dimR = n$, then R is a G-type domain if and only if the number of nonzero minimal prime ideals in R are countable. **Proof.** By Theorem "2.2" it is trivial. Let R be a noetherian domain with the CPA property, then R is a G-type domain if and only if Spec(R) is countable and each nonzero prime ideal is maximal(i.e.,$k-dimR \leq 1$).

If R be a G-type domain, then $Spec(\widetilde{R})$ is homeomorphic to $Spec(R)$ (via the map induced by the natural inclusion of R in $\widetilde{R}$).

If R is a Prüfer G-type domain, then $S^{-1}R \subset R^*$, R has pullback type, and $R^* = \cap\{R_P|P \in Spec^1(R)\}$ has essential type. In addition, if R is a Bézout G-type domain, then $S^{-1}R = R^*$.

It is exhibited a one-dimensional quasilocal domain R such that R^* is a one-dimensional (therefore, essential) Prüfer G-type domain, but not semiquasilocal. Let V be a one-dimensional valuation domain with quotient field K such that there exists an algebraic field extension L of K having infinitely many valuation subrings extending V. (for instance, take $V = Z_{PZ}$ and L the field of algebraic numbers.) Let T be the integral closure of V in L. Then T is one-dimensional and Prüfer, but not semiquasilocal.

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A NOTE ON SOME TOP LOCALCOHOMOLOGY MODULES

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Abstract: In this paper we examine $Att_R(H^{d-1}_{I,J}(R))$ in which $d := dim(R)$, the set of attached prime ideals of $H^{d-1}_{I,J}(R)$. As a technical tool we use the concept of colocalization introduced by Richardson in [3].
Keywords: Top local cohomology module, colocalization, attached prime ideal.

1 INTRODUCTION

Local cohomology modules with respect to a pair of ideals was introduced firstly in [4].

Recall that for an R-module K, a prime ideal p of R is said to be an attached prime ideal of K if $p = \mathrm{Ann}(K/N)$ for some submodule N of K. We denote the set of attached prime ideals of K by $\mathrm{Att}_R(K)$. When K has a secondary representation, this definition agrees with the usual definition of attached primes. If K is an Artinian R-module, so that K admits a reduced secondary representation $K = K_1 + \cdots + K_r$ such that K_i is p_i-secondary, $i = 1, \cdots, r$, then $\mathrm{Att}_R(K) = \{p_1, \cdots, p_r\}$ is a finite set. M. Eghbali in [1] examine $\mathrm{Att}_R(H^{d-1}_a(R))$ in which $d := \dim(R)$ and a is an ideal of R.

Assume that (R, m) is a local ring and S a multiplicative closed subset of R. Recently, A. S. Richardson [2] has proposed the definition of colocalization of an R-module M relative to S as the $S^{-1}R$-module $S_{-1}M = D_{S^{-1}R}(S^{-1}D_R(M))$, where $D(-)$ stands for the Matlis duality functor. In the light of [2, Theorem 2.2], representable modules are preserved under colocalization and in case M is a representable module, we have $\mathrm{Att}_{S^{-1}R}S_{-1}M = \{S^{-1}p : p \in \mathrm{Att}M \text{ and } S \cap p = \emptyset\}$. (1)

In this paper we examine $\mathrm{Att}_R(H^{d-1}_{I,J}(R))$ the set of attached prime ideals of $H^{d-1}_{I,J}(R)$ in which $d = \dim(R)$. As a technical tool we use the concept of colocalization introduced in [2].

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2 ATTACHED PRIME IDEALS OF $H^{d-1}_{I,J}(R)$

Assume that $(R, m)$is a local ring with $d = \dim(R)$.

Theorem 2.1. *Let c be an integer such that $H^c_{I,J}(R)$ is a representable R-module and $H^i_{I,J}(R) = 0$ for every $i > c$. Then*

(i) $\mathrm{Att}_{R_p}({}^pH^c_{I,J}(R)) \subseteq \{qR_p : q \in \mathrm{Spec}(R), q \subseteq p \text{ and } \dim R/q + J \geq c\}$, *for all* $p \in \mathrm{Spec}(R)$.

(ii) $\mathrm{Att}_{R_p}({}^pH^{\dim R}_{I,J}(R)) = \{qR_p : J \subseteq q \subseteq p, \sqrt{I+q} = m, \dim R/q = \dim R\}$.

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Proof. (i) Assume that $qR_p \in \mathrm{Att}_{R_p}({}^pH^c_{I,J}(R))$. Then $q \in \mathrm{Spec}(R)$ and $q \subseteq p$ also by virtue of (1), $q \in \mathrm{Att}H^c_{I,J}(R)$. Hence, $q \in \mathrm{Att}H^c_{I,J}(R/q)$. This implies that $H^c_{I,J}(R/q) \neq 0$ and consequently $\dim R/q + J \geq c$, by [4, Theorem 4.3].

Let $qR_p \in \mathrm{Att}_{R_p}({}^pH^d_{I,J}(R))$. As we have seen in part one $H^d_{I,J}(R/q) \neq 0$ so $\dim R/q = d$ and by Hartshorn Litchenbaum Vanishing Theorem $\sqrt{I+q} = m$ and $J \subseteq q$ so it is enough to show that $q \in \mathrm{Att}H^d_{I,J}(R)$. As $\dim R/q = d$ so Independence Theorem implies that $H^c_{I,J}(R/q) \neq 0$. Hence easily one can see that

$$\emptyset \neq \mathrm{Att}H^c_{I,J}(R/q) = H^c_{I,J}(R)) \cap \mathrm{Supp}(R/q). \quad (2)$$

In the contrary assume that $q \notin \mathrm{Att}H^d_{I,J}(R)$. Then by virtue of (2) there exists a prime ideal $q_0 \in \mathrm{Att}H^d_{I,J}(R)$ such that $q_0 \supseteq q$ and so $\dim R/q_0 < d$. On the other hand $q_0 \in \mathrm{Att}H^d_{I,J}(R)$ if and only if $q_0R_{q_0} \in \mathrm{Att}_{R_{q_0}}({}^{q_0}H^d_{I,J}(R))$. By virtue of part one $\dim R/q_0 = d$ which is a contradiction. Now the proof is complete. □

Theorem 2.2. *Let R be a complete local ring of dimension d. Let I, J be two ideals of R. Assume that $H^{d-1}_{I,J}(R)$ is representable and $H^d_{I,J}(R) = 0$. Then*

(i) $\mathrm{Att}H^{d-1}_{I,J}(R) \subseteq \{p \in \mathrm{Spec}(R) : \dim R/p = d-1, J \subseteq p, \sqrt{I+p} = m\} \cup \mathrm{Assh}(R)$.

(ii) $\{p \in \mathrm{Spec}(R) : \dim R/p = d-1, J \subseteq p, \sqrt{I+p} = m\} \subseteq \mathrm{Att}H^{d-1}_{I,J}(R)$.

Proof. (i): Let $p \in \mathrm{Att}H^{d-1}_{I,J}(R)$, then $pR_p \in \mathrm{Att}^p_{R_p}H^{d-1}_{I,J}(R)$. Hence by Theorem2.1 $\dim R/p \geq d-1$ and $J \subseteq p$. When $\dim R/p = d$ it follows that $p \in \mathrm{Assh}(M)$. In the case $\dim R/p = d-1$, as $p \in \mathrm{Att}H^{d-1}_{I,J}(R)$ and $H^{d-1}_{I,J}(-)$ is a right exact functor so one can deduce that $H^{d-1}_{I,J}(R/p) \neq 0$.

(ii) Let $\dim R/p = d-1$ with $\sqrt{I+p} = m$ and $J \subseteq p$. Then Theorem 2.1(ii) implies that $pR_p \in \mathrm{Att}^p_{R_p}H^{d-1}_{I,J}(R/p)$. By (1) we deduce that $p \in \mathrm{Att}H^{d-1}_{I,J}(R/p)$. On the other hand the epimorphism

$$H^{d-1}_{I,J}(R) \to H^{d-1}_{I,J}(R/p) \to 0$$

implies that $p \in \mathrm{Att}H^{d-1}_{I,J}(R)$. □

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On Gamma S-acts

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Abstract: In this paper, using the notion of "Γ-semigroups", we study action over Γ-semigroups as a generalization of action over semigroups, and investigate basic properties and classical notions and results of acts a generalize to gamma acts.

Keywords: Γ-act, Γ-homomorphism, Γ-congruence, Γ-ideal.

1 INTRODUCTION

The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. A semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation.

In 1981, Sen and Later, In 1986, Senan Saha introduced the notion of the Γ-semigroup, which is in facta generalization of the notation of the semigroup. The topic of investigations about Γ-semigroups have been analyzed by a lot of mathematicians for instance by Sen, Saha, Qutta, Adhikari, Hila,... .

Many classical notions and results of the theory of semigroup have been extended a generalized to Γ-semigroups by a lot of mathematicians. Action over semigroups S-acts were introduced by Hoehnke. One of the very useful notions in many branches of mathematics as well as in computer science is the action of a semigroup or a monoid on a set.

In this paper, we introduce action over Γ-semigroups as a generalization of action over semigroups and investigate classical notion and results of the theory of acts extend a generalis to Γ-acts. So we generalized the notions:subacts, fixed element, identity, homomorphism, congruences, quotionts and homomorphism theorem.

In section 1, the notion of a Γ-act is introduce and some examples are given. The concept of gamma acts of a Γ-semigroup has been introduced and some basic properties and have been obtained. Finally we show investigate the essential homomorphism theorem in Γ-acts.

In the following we first recall some facts about the category $S - Act$ needed in this paper.

Let S be a semigroup and A be a set. If we have a mapping (called the actio of S on A)

$$\mu : S \times A \to A$$

$$(s, a) \mapsto sa := \mu(s, a) \qquad (1)$$

Such that $(st)a = s(ta)$ for $a \in A,\ s, t \in S$ we call A a (left) $S-act$ or a (left) act over S and write ${}_SA$.

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If S is a monoid with identity 1, we usually also require that $1a = a$ for $a \in A$.

Let S and Γ be non-empty sets we call S to be a Γ-semigroup if there exists a mapping $S \times \Gamma \times S \to S$ writing (a, γ, b) by $a\gamma b$, such that S satisfies the identity $(a\gamma b)\beta c = a\gamma(b\beta c)$ for all $a, b, c \in S$ ad $\gamma, \beta \in \Gamma$.

An element a of a Γ-semigroup S is said to be:

(i) A Left identity of S provided $(a\gamma s) = s$ for all $s \in S$ and $\gamma \in \Gamma$.

(ii) A right identity of S provided $(s\gamma a) = s$ for all $s \in S$ and $\gamma \in \Gamma$.

(iii) An identity provided it is both a left identity and right identity of S.

A Γ-semigroup with identity is called a Γ-monoid.

If S is a Γ-semigroup, so a mapping

$$S \times \Gamma \times A \to A$$

$$(s, \gamma, a) \mapsto s\gamma a = a$$

Makes A to be a $\Gamma - S$-act.

2 On Gamma S-acts

Definition 2.1. Let S be a Γ-semigroup and A be a set. If we have a mapping

$$\lambda : S \times \Gamma \times A \to A$$

$$(s, \gamma, a) \mapsto s\gamma a := \lambda(s, \gamma, a)$$

such that $(s\gamma t)\beta a = s\gamma(t\beta a)$ for $a \in A$, $s, t \in S$ and $\gamma, \beta \in \Gamma$, we call A a (left) $\Gamma - S$-act or a (left) Γ-act over S, and write $\Gamma -_S A$.

If S is a Γ-monoid with identity 1, we usually also require that $1\gamma a = a$ for $a \in A$ and $\gamma \in \Gamma$.

Therefor if $S = \{1\}$ such that 1 is identity of S then every non-empty set A is a left and right $\Gamma - S$-act.

Lemma 2.2. Let S be a Γ-semigroup, then S on itself is a $\Gamma - S$-act.

Example 2.3. Let S be the set of all 3×2 matrices over Z, the set of integers numbers and Γ be the set of all 2×3 matrices over Z and M be the set of all 3×3 matrices over Z. Define $A\gamma B$=usual matrix product of A, γ, B, for all $A \in S$, $\gamma \in \Gamma$ and $B \in M$. Then M is a $\Gamma - S$-act, Note that M is not S-act.

Example 2.4. Let A is a $\Gamma - S$-act and γ a fixed element of Γ. We define $s.a = s\gamma a$ for every $s \in S$ and $a \in A$. We can show that A with this action is a S-act.

Example 2.5. Let A is a S-act and Γ be a non-empty set. Define a mapping from $S \times \Gamma \times A$ to A as, $s\gamma a = sa$ for every $s \in S$, $\gamma \in \Gamma$ and $a \in A$. Then A is a $\Gamma - S$-act.

Note that every S-act can be considered to be a $\Gamma - S$-act. Thus the class of all $\Gamma - S$-acts includes the class of all S-acts.

Note that $\Gamma - S$-act is generalization of a S-act.

Definition 2.6. Let A be a $\Gamma - S$-act and $A' \subseteq A$ a non-empty subset. Then A' is called a Γ-subact of A if $s\gamma a' \in A'$ for all $s \in S$, $a' \in A'$ and $\gamma \in \Gamma$. (or $S\Gamma A' \subseteq A'$).

Definition 2.7. Let A be a Left $\Gamma - S$-act. A element $\theta \in A$ is called a right zero of A (a fixed element) if $s\gamma\theta = \theta$ for every $s \in S$ and $\gamma \in \Gamma$. Note that $\{\theta\}$ is a Γ-subact.

An $\Gamma - S$-act can have more than one zero, for example:

Lemma 2.8. If the Γ-monoid S has a right zero z, then every element $z\gamma a, a \in A$ and $\gamma \in \Gamma$ is a right zero of A.

Definition 2.9. We call A a simple Γ-act if it contains no Γ-subacts other than A itself.

We call A a θ-simple Γ-act if it contains no other than A and one 1-element Γ-subact, i.e. $_S\theta = \{\theta\}$.

Lemma 2.10. Let S and U be Γ-semigroup. Then the product of $S \times U = T$ is a Γ-semigroup.

3 Homomorphism Theorem for $\Gamma - S$-Acts

Definition 3.1. Let $_SA,_SB$ be two left $\Gamma - S$-acts. A mapping $f :_S A \to_S B$ is called a Γ-homomorphism of left $\Gamma - S$-acts or just an $\Gamma - S$-homomorphism, if:

$f(s\gamma a) = s\gamma f(a)$ for every $a \in_S A$, $s \in S$ and $\gamma \in \Gamma$.

The set of all $\Gamma - S$-homomorphism from $_SA$ in $_SB$ will be denoted by $Hom(_SA,_SB)$ or sometimes by $Hom_S(A, B)$ (that S is Γ-semigroup). Clearly, $id_A :_S A \to_S A$ is a Γ-homomorphism.

A $\Gamma - S$-homomorphism $f :_S A \to_S B$ is called an $\Gamma - S$-isomorphism if f is bijective and write $_SA \cong_S B$.

Note that if $f :_S A \to_S B$ be $\Gamma - S$-homomorphism therefore $f(_SA)$ is Γ-subact of $_SB$.

Lemma 3.2. The composition gf of Γ-homomorphisms $f :_S A \to_S B$, $g :_S B \to_S C$ of left $\Gamma - S$-acts is a Γ-homomorphism, i.e. $gf \in Hom_S(A, C)$.

Lemma 3.3. The inverse mapping f^{-1} of a bijective Γ-homomorphism f of left $\Gamma - S$-acts is a Γ-homomorphism of left $\Gamma - S$-acts.

Definition 3.4. Let $_SA$ be a $\Gamma - S$-act. An equivalence relation ρ on A is called a $\Gamma - S$-act congruence or a Γ-congruence on $_SA$, if $a\rho a'$ implies $(s\gamma a)\rho(s\gamma a')$ for $a, a' \in_S A$, $s \in S$ and $\gamma \in \Gamma$.

Definition 3.5. Let $_SA$ be a $\Gamma - S$-act and ρ be a Γ-congruence on $_SA$. Then $\frac{_SA}{\rho} = \{[a]_\rho | a \in_S A\}$ with $s\gamma[a]_\rho = [s\gamma a]_\rho$ for every $s \in S$ and $\gamma \in \Gamma$ is called factor Γ-act of $_SA$ by ρ. Note that, If S is a Γ-monoid then any left Γ-congruence ρ on S is a Γ-act congruence on $_SS$.

So $\frac{_SS}{\rho} = \{[s]_\rho | s \in_S S\}$ such that for every $[s]_\rho \in \frac{_SS}{\rho}$ we have $[s]_\rho = [s\gamma 1]_\rho = s\gamma[1]_\rho$ for every $\gamma \in \Gamma$, therefore $\frac{_SS}{\rho}$ is generated by $[1]_\rho$ such that 1 is identity of S we denote $\frac{_SS}{\rho} =< [1]_\rho >$.

Lemma 3.6. Let ρ be a Γ-congruence on a Γ-act $_SA$. The canonical surjection

$$\Pi_\rho :_S A \to \frac{_SA}{\rho}$$

$$a \mapsto [a]_\rho$$

is a Γ-homomorphism. It is called a canonical epimorphism.

Definition 3.7. Let $_SB$ be Γ-subact of $_SA$. An equivalence relation ρ_B on $_SA$ is called a Γ-Rees congruence on $_SA$, if $a\rho_B a'$ implies $a, a' \in B$ or $a = a'$ for $a, a' \in_S A$.

Definition 3.8. If ${}_SB$ be Γ-subact of ${}_SA$ Rees factor $\frac{{}_SA}{{}_SB} = \{[a]\rho_B | a \in_S A\}$ with Γ-Rees congruence ρ_B on A.

Definition 3.9. Let S be Γ-semigroup and K be subset of S. K is called a left Γ-ideal of S if $S\Gamma K = \{s\gamma K | s \in S, \gamma \in \Gamma \; and \; k \in K\}$ be subset of K. In particular Γ-ideal K from S is Γ-subact of S.

Corollary 3.10. If ${}_SK$ is a left Γ-ideal of ${}_SS$ then $\frac{{}_SS}{{}_SK}$ is the Γ-Rees factor of ${}_SS$ by the left Γ-ideal ${}_SK$.

Definition 3.11. Let $f :_S A \rightarrow_S B$ be Γ-homomorphism, $\rho = kerf$ that $a\rho a' \Leftrightarrow f(a) = f(a')$ then ρ is called kernel Γ-congruence of f.

Theorem (Homomorphism theorem for Γ-acts)3.12. Let $f :_S A \rightarrow_S B$ be a Γ-homomorphism and ρ be a Γ-congruence of ${}_SA$ such that $a\rho a'$ implies $f(a) = f(a')$, i.e. $\rho \leq ker \; f$. Then $f' : \frac{{}_SA}{\rho} \rightarrow_S B$ with $f'([a]_\rho) := f(a)$, $a \in_S A$, is the unique Γ-homomorphism such that the following diagram is commutative. $(\Pi_\rho :_S A \rightarrow \frac{{}_SA}{\rho})$

${}_SA \longrightarrow_S B$

$\searrow \frac{{}_SA}{\rho} \nearrow$

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Armendariz Ideals Relative to a Monoid

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Abstract: For a monoid M, we introduce M-Armendariz ideals, which are generalization of Armendariz ideals, and investigate some results involving theme. We show that if R be a right Ore ring with classical right quotient ring Q and I is M-Armendariz left ideal of R, then QI is M-Armendariz left ideal of Q.
Keywords: α-irreducible, α-strongly irreducible, faithful module, multiplication module.

1 INTRODUCTION

Throughout this article R denotes an associative ring with identity, M denotes a monoid with identity e. The left annihilator of a subset A of a ring R is denote by $r_R(A)$ or $r(A)$. Rege and Chhawchharia [1] introduced the notion of an Armendariz ring. They define a ring R to be an Armendariz ring if whenever polynomials $f(x) = a_0+a_1x^1+\cdots+a_mx^m$, $g(x) = b_0+b_1x^1+\cdots+b_nx^n \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j = 0$ for each i and j. (The converse is always true.). A monoid M is called a $u.p.$-monoid (unique product monoid) if for any two non empty finite subsets $A, B \subseteq M$, there exists an element $g \in M$ uniquely presented in the form ab where $a \in A$ and $b \in B$. Liu [3] called a ring R is M-Armendariz if whenever elements $\alpha = a_1g_1+\cdots+a_mg_m$, $\beta = b_1h_1+\cdots+b_nh_n \in R[M]$ satisfy $\alpha\beta = 0$, then $a_ib_j = 0$ for all i, j. Which is a generalization of Armendariz rings. We recall that a left ideal I of R is called Armendariz [2] if whenever polynomials $f(x) = a_0 + a_1x^1 + \cdots + a_mx^m$, $g(x) = b_0 + b_1x^1 + \cdots + b_nx^n \in R[x]$ satisfy $f(x)g(x) \in r_{R[x]}(I[x])$, then $a_ib_j \in r_R(I)$ for each i and j. In this article a left ideal I of R is called M-Armendariz if whenever elements $\alpha = \sum_{i=1}^{n} a_ig_i$, $\beta = \sum_{j=1}^{m} b_jh_j \in R[M]$ satisfy $\alpha\beta \in r_{R[M]}(I[M])$ we have $a_ib_j \in r_R(I)$ for all i, j. Which is a generalization of Armendariz ideals. Let $M = \{\mathbb{N} \cup \{0\}, +\}$. Then a ring R is M-Armendariz if and only if R is Armendariz. Also, if $M = \{\mathbb{N} \cup \{0\}, +\}$, then a ideal I of ring R is M-Armendariz if and only if I is Armendariz.

2 Main Results

A left ideal I of R is called abelian if for each idempotent elements $e \in R$, $er - re \in r_R(I)$ for any $r \in R$.

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Definition. Let R be a ring.

(a) A left ideal I of R is called M-Armendariz if whenever elements $\alpha = \sum_{i=1}^{n} a_i g_i, \beta = \sum_{j=1}^{m} b_j h_j \in R[M]$ satisfy $\alpha\beta \in r_{R[M]}(I[M])$ we have $a_i b_j \in r_R(I)$ for all i, j.

(b) A left ideal I of R is called reduced if $a^2 \in r_R(I)$, we have $a \in r_R(I)$ for every $a \in R$.

Clearly, the converse assertion of the condition of definition holds for every ring R and every left ideal I of R since $r_{R[M]}(I[M]) = r_R(I)[M]$.

A classical right ring for a ring R is a right of fractions for R with respect to set of all regular elements in R. The situation when R has a classical right quotient right Q is a also denoted by saying that R is a right r, $s \in R$ such that $xr = ys \neq 0$. A ring R is called right Ore if for each nonzero $x, y \in R$ there exist $r, s \in R$ such that $xr = ys \neq 0$. A ring R has a classical right quotient ring if and only if the set of regular elements in R is right Ore set

Theorem 2.1. *Let M be a monoid and R be a right Ore ring with classical right quotient ring Q. If I is M-Armendariz left ideal of R, then QI is M-Armendariz left ideal of Q.*

We know that any subring of an M-Armendariz ring is M-Armendariz ring. In the following proposition we prove that any left ideal of R is an M-Armendariz left ideal provided that R is M-Armendariz ring.

Theorem 2.2. *If R is an M-Armendariz ring, then each left ideal of R is an M-Armendariz left ideal.*

Let R be a ring. Define a ring $T(R)$ as follows:

$$\left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} | a, b, c, d \in R \right\}.$$

For a u.p.-monoid M we give an example of a nonzero M-Armendariz left ideal of a non-M-Armendariz ring.

Example. Let $R = \mathbb{Z}_4$, M a u.p.-monoid and $S = T(R)$. Since the ring R is not reduced, by [3, Propositions 1.1 and 1.7], S is not $M-$Armendariz. Write $a = \begin{pmatrix} 0 & \overline{2} & \overline{2} \\ 0 & 0 & \overline{2} \\ 0 & 0 & 0 \end{pmatrix}$, $I = Sa$ and $r(I) = r_s(I)$, then

$$r(I) = \left\{ \begin{pmatrix} r & a & b \\ 0 & r & c \\ 0 & 0 & r \end{pmatrix} | r \in \{0, \overline{2}\}, a, b, c \in \mathbb{Z}_4 \right\}.$$

For the upper triangular matrix ring $T(R)$, we have $nil(T(R)) = \begin{pmatrix} nil(R) & R & R \\ 0 & nil(R) & R \\ 0 & 0 & nil(R) \end{pmatrix}$, therefore $nil(T(R)) = r(I)$. So $S/r(I)$ is a reduced ring, hence I is an M-Armendariz left ideal of S.

Theorem 2.3. *Let M be a u.p.-monoid and I a reduced left ideal. Then I is an M-Armendariz left ideal.*

Theorem 2.4. *Let M be a monoid and N a u.p.-monoid. If I is a reduced left ideal and M-Armendariz left ideal, then $I[N]$ is M-Armendariz left ideal.*

Theorem 2.5. *Let M be a monoid and N be a u.p.-monoid. If I is a reduced and M-Armendariz left ideal, then I is $M \times N$-Armendariz left ideal.*

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ایده‌آل‌های استلزامی ضعیف تولید شده توسط یک زیر مجموعه از جبر استلزامی مشبکه

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چکیده: جبرهای استلزامی مشبکه یک ساختار جبری منطقی هستند که براساس ترکیب شبکه و جبر استلزامی بنا شده‌اند. در این مقاله قصد داریم به اصطلاح یک تعریف و اثبات دو قضیه در رابطه با نماد LI -ایده‌آل‌های ضعیف در جبرهای استلزامی مشبکه، که توسط J.J. Lai ، در مقاله ای با عنوان "ایده آل‌های استلزامی ضعیف در جبرهای استلزامی مشبکه"(۲۰۰۶) و M. Jun ، Y. Xu ، J.J. Lai در مقاله با عنوان "توسیعی از LI -ایده‌آل‌ها در جبرهای استلزامی مشبکه" (۲۰۰۷) به تحریر درآمده است، بپردازیم.

کلمات کلیدی: جبر استلزامی مشبکه، LI -ایده‌آل، WLI -ایده آل.

مقدمه

اخیرا منطق غیر کلاسیک یک ابزار مفید برای هوش مصنوعی بوده است. چون منطق چند مقداری توسیع بزرگ و پیشرفته‌ای از منطق کلاسیک است لذا در طراحی و اثبات سیستم‌ها، روش جالبی نسبت به منطق کلاسیک فراهم می کند. در حوزه منطق چند مقداری، مشبکه نقش مهمی را ایفا می کند.

از اینرو گاگون (Goguen)، پاولکا (Pavelka)، نواک (Novak) روی دستگاه فرمولی منطق مقداری مشبکه تحقیق کردند بعلاوه به خاطر تحقیق در دستگاه‌های منطق چند مقداری که ارزش گزاره ایشان در مشبکه داده شده است، در سال ۱۹۹۰، یانگ (Xu Yang) نظریه جبرهای استلزامی مشبکه را ارایه کرد و خاصیت های زیادی را در مورد آنها اثبات کرد. از آن پس این جبر منطقی به طور وسیع مورد مطالعه محققان قرار گرفت. در مقاله جون (Jun) مفهوم LI -ایده‌آلها در جبرهای استلزامی مشبکه را تعریف کرده وخاصیتهای آنها را مورد بررسی قرار داده است. برای توسعه کلی جبرهای استلزامی مشبکه، نظریه ایده آلها نقش مهمی را در جبرهای استلزامی مشبکه ایفا می کند. در این مقاله به عنوان تعمیمی از نظریه ایده آل ها، مفهوم WLI -ایده آل ها در جبرهای استلزامی مشبکه را معرفی و چند خاصیت وقضیه آن را اثبات می کنیم.

جبرهای استلزامی مشبکه[۱]

تعریف ۱. *مشبکه کراندار* $(L, \vee, \wedge, ', o, I)$ *که در آن «* $'$ *» یک عمل یکانی جذبی معکوس مرتب، همراه با*

[۱] *Lattice Implication Algebras*

عمل دو تائی « $\to$ » که برای هر $x, y, z \in L$ در شرایط زیر صدق کند را یک جبر استلزامی مشبکه می نامند.

$L_1) x \to (y \to z) = y \to (x \to z);$

$L_2) x \to x = I;$

$L_3) x \to y = y' \to x';$

$L_4) \ x \to y = y \to x = I \Rightarrow \ x = y;$

$L_5) (x \to y) \to y = (y \to x) \to x;$

$L_6) (x \vee y) \to z = (x \to z) \wedge (y \to z);$

$L_7) \ (x \wedge y) \to z = (x \to z) \vee (y \to z).$

اگر رابطه « $\leq$ » روی جبر استلزامی مشبکه L به صورت زیر تعریف شود:

$x \leq y \Leftrightarrow x \to y = I.$

آنگاه جبر استلزامی مشبکه L همراه با رابطه « $\leq$ » به یک مجموعه جزئاً مرتب تبدیل می‌شود.

WLI-ایده‌آل‌ها و خواص آن در جبرهای استلزامی مشبکه

تعریف ۲. *فرض کنید در جبر استلزامی مشبکه L برای هر $x, y \in L$ اعمال دوتایی $\otimes$ و $\oplus$ چنین تعریف شود:*

$$x \otimes y = (x \to y')' \quad , \quad x \oplus y = x' \to y$$

گزاره ۱. *در جبر استلزامی مشبکه L برای هر $x, y, z \in L$، روابط زیر برقرارند:*

۱) $x \otimes y = y \otimes x, x \oplus y = y \oplus x;$

۲) $x \oplus x' = I$ ، $x \otimes x' = 0$ ، $x \oplus I = I$

$x \otimes I = x$ ، $x \oplus 0 = x$ ، $x \otimes 0 = 0;$

۳) $(x \otimes y)' = x' \oplus y', (x \oplus y)' = x' \otimes y';$

تذکر: در یک جبر استلزامی مشبکه L برای هر $x, y \in L$، $x \oplus y = 0$ اگر و تنها اگر $x = y = 0$ و $x \otimes y = I$ اگر و تنها اگر $x = y = I$.

تعریف ۳. *فرض کنید L یک جبر استلزامی مشبکه باشد. زیر مجموعه غیرتهی A از L را LI-ایده‌آل ضعیف یا به اختصار WLI-ایده‌آل L نامند، هرگاه برای هر $x, y \in L$، $x \otimes y' \in A$ نتیجه دهد*

$(x \otimes y') \otimes y' \in A$

قضیه ۱. *فرض کنیدL یک جبر استلزامی مشبکه باشد. و برای مجموعه اندیس گذار J، خانواده، $\{A_i | i \in J\}$ WLI-ایده‌آل‌های L باشد. آنگاه $\bigcup_{i \in J} A_i$ و $\bigcap_{i \in J} A_i$، WLI-ایده‌آل L می‌باشند.*

اما J.J. Lai در مقاله خود با عنوان "LI-ایدال‌های استلزامی ضعیف در جبرهای استلزامی مشبکه" WLI-ایدال تولید شده توسط یک زیر مجموعه A از جبر استلزامی مشبکه L را، بزرگترین WLI-ایدال شامل A می‌نامد، که این تعریف باید به صورت زیر اصلاح شود:

تعریف ۴. *فرض کنید L یک جبر استلزامی مشبکه باشد و $A \subseteq L$ ، کوچکترین WLI-ایده‌آل L شامل A را WLI-ایده‌آل تولید شده، توسط A نامند و آنرا با $< A >_L$ نشان می‌دهند.*

در ادامه برای هر $a \in L$ تعریف می‌کنیم:

$L_a^1 := \{((x \to y)' \to y)' \mid \ x, y \in L, (x \to y)' = a\}$

$L_a^2 := \{((x \to y)' \to y)' \mid x, y \in L, (x \to y)' \in L_a^1\} = \{(x \otimes y') \otimes y' \mid x, y \in L, x \otimes y' \in L_a^1\}.$

$L_a^3 := \{((x \to y)' \to y)' \mid x, y \in L, (x \to y)' \in L_a^2\} = \{(x \otimes y') \otimes y' \mid x, y \in, x \otimes y' \in L_a^2\}.$

$L_a^n := \{((x \to y)' \to y)' \mid x, y \in L, (x \to y)' \in L_a^{n-1}\} = \{(x \otimes y') \otimes y' \mid x, y \in, x \otimes y' \in L_a^{n-1}\}.$

J.J. Lai در همین مقاله مجموعه

$L_a^n := \{((x\to y)'\to y)' \mid x,y\in L, (x\to y)'\in L_a^{n-1}\}$

را تعریف می کند ولی در ادامه رابطه غلط جزیت زیر را نتیجه می گیرد:

$$L_a^n \subseteq L_a^{n-1} \subseteq ... \subseteq L_a^3 \subseteq L_a^2 \subseteq L_a^1$$

و اشتراک آنها را $T_a=\cap_{i=1}^{\infty} L_a^i$ می‌نامد ودر قضیه ای نشان می‌دهد T_a یک WLI-ایده‌ال است. که این رابطه جزیت باید به صورت زیر اصلاح شود:

$$...L_a^n \supseteq L_a^{n-1} \supseteq ... \supseteq L_a^3 \supseteq L_a^2 \supseteq L_a^1$$

و اثبات همان قضیه به صورت زیر تصحیح شود:

از اینرو:

$$L_a^1 \subseteq L_a^2 \subseteq ... \subseteq L_a^i \subseteq L_a^{i+1} \subseteq ...$$

زیرا فرض کنید برای $i\ge 1$، $((x\to y)'\to y)' = L_a^i$ در اینصورت:

(۱)

$$((x\to y)'\to y)' = (((x\to y)'\to y)'\to o)'$$

(۲)

$$((x\to y)'\to y)' = ((((x\to y)'\to y)'\to o)'\to o)'$$

اکنون با اختیار $((x\to y)'\to y)' = X$ بنا به (۱)، $(X\to o)'\in L_a^i$ و در نتیجه بنا بر تعریف L_a^{i+1} خواهیم داشت، $((X\to o)'\to o)'\in L_a^{i+1}$ از طرفی بنابر (۲)، $((X\to o)'\to o)' = ((x\to y)'\to y)'$ بنابراین، $((x\to y)'\to y)'\in L_a^{i+1}$. لذا $L_a^i\subseteq L_a^{i+1}$.
همچنین اجتماع L_a^i ها $(i\ge 1)$ را با T_a نشان میدهند.
به عبارت دیگر $T_a=\cup_{i=1}^{\infty} L_a^i$ اجتماع L_a^i ها $(i\ge 1)$ را با T_aنشان میدهند. به عبارت دیگر $T_a=\cup_{i=1}^{\infty} L_a^i$

قضیه ۲. *اگر L یک جبر استلزامی مشبکه باشد. آنگاه برای هر $a\in L$، T_a یک WLI-ایده‌آل است*

اثبات. فرض کنید برای $x,y\in L$، $(x\otimes y')\in T_a$. در این صورت $i\ge 1$ وجود دارد که $i\in\{0,1,2,...\}$ ، $(x\otimes y')\in L_a^i$. لذا $(x\otimes y')\otimes y'\in L_a^{i+1}$، برای یک $i\in\{1,2,...\}$.
به عبارت دیگر $(x\otimes y')\otimes y'\in T_a$. بنابراین T_a یک WLI-ایده‌آل L است.
همچنین در همین مقاله صورت قضیه زیر و اثبات آن:
”فرض کنید L یک جبر استلزامی مشبکه بوده و $A\subseteq L$ آنگاه $<A>=\bigcap_{a\in A}(a)$ “ باید به صورت زیر تصحیح شود:

قضیه ۳. *فرض کنید L یک جبر استلزامی مشبکه و $A\subseteq L$ آنگاه $<A>=\bigcup_{a\in A}<a>$.*

اثبات. چون برای هر $a\in A$ ، $a\in <a>$ لذا $A\subseteq \bigcup_{a\in A}<a>$ و چون اجتماع WLI- ایده‌آل ها نیز، WLI- ایده‌آل است. بنابراین $<A>\subseteq \bigcup_{a\in A}<a>$.
زیرا $<A>$، کوچکترین WLI-ایده‌آل شامل A است.
از طرف دیگر برای هر $a\in A$، $<a>\subseteq <A>$. لذا $\bigcup_{a\in A}<a>\subseteq <A>$. بنابراین $<A>=\bigcup_{a\in A}<a>$.

نتیجه گیری

همانطور که میدانیم ایده آل ها با ویژگیهای خاصشان نقش مهمی را در ساختار دستگاه منطقی ایفا می کنند . لذا مطالعه مفهوم *WLI* - ایده آل وبررسی ویژگیهای آن منجربه پیشبرد سیستم منطقی با ارزش گزاره ای خواهد شد . از اینرو این مقاله اشتباهات دو قضیه مهم:

۱) اگر L یک جبر استلزامی مشبکه باشد. آنگاه برای هر $a\in L$، T_a یک WLI-ایده‌آل است.

۲) فرض کنید L یک جبر استلزامی مشبکه و $A\subseteq L$ آنگاه

$< A >= \bigcup_{a \in A} < a >$

که توسط J.J. Lai در مقاله "WLI-ایده آل ها در *جبرهای استلزامی مشبکه*" و M. Jun ، Y. Xu در مقاله " *توسیعی از LI- ایده آل ها در جبرهای استلزامی مشبکه*" به تحریر در آمده است را باز نویسی کرده است.

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On Self-Auto-Permutable Subgroups of a Finite Group

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Abstract: Let G be a finite group. A subgroup H of G is called self-auto-permutable in G if $HH^{\alpha} = H^{\alpha}H$ implies $H^{\alpha} = H$ for $\alpha \in Aut(G)$. In this paper, we study the influence of self-auto-permutable on the structure of a finite group.
Keywords: Finite group, auto-permutable, self-auto-permutable, Weakly characteristic, subcharacteristic condition.

1 INTRODUCTION

self-conjugate-permutable subgroup of a group G was introduced by Z. Shen in [4]. A subgroup H of G is said to be self-conjugate-permutable if $HH^x = H^xH$ implies $H^x = H$ for any $x \in G$. In this paper we extend this definition and introduce a new subgroup of a group G named self-auto-permutable of a group G. H a subgroup H of G is called self-auto-permutable of a group G if $HH^{\alpha} = H^{\alpha}H$ implies $H^{\alpha} = H$ for $\alpha \in Aut(G)$. It is clear if we consider $\alpha \in Inn(G)$ then self-auto-permutablity and self-conjugate-permutablity are equal. Here all groups are finite. In this paper we study the influence of self-auto-permutable on the structure of a group.

2 Main Results

Subgroup H of G be auto-permutable if $HH^{\alpha} = H^{\alpha}H$ for any $\alpha \in Aut(G)$.
We start with the following lemmas.

Definition 2.1. *A subgroup H of a group G is said to be self-auto-permutable subgroup, if for each $\alpha \in Aut(G)$ the equality $HH^{\alpha} = H^{\alpha}H$, implies that $H = H^{\alpha}$.*

Lemma 2.2. *Let H be a subgroup of G. Then we have:*
i) H^cG if and only if HK^cG and H self-auto-permutable subgroup in G.
ii) H^cG if and only if H is a self-auto-permutable and auto-permutable in G

Lemma 2.3. *Let G be a group. Suppose that H is self-auto-permutable in G, $K \leq G$ and N^cG. Then we have:*
i) Let $N \leq K$. Then $\frac{K}{N}$ is self-auto-permutable in $\frac{G}{N}$ and K is self-auto-permutable in G.
ii) If K is a p-subgroup of G with $(\mid K \mid, \mid N \mid) = 1$, then K is self-auto-permutable in G if and only if KN is self-auto-permutable in G.

Lemma 2.4. *Let H be a self-auto-permutable subgroup of a group G, with $Aut(G)$ being abelian. Then H^{α} is self-auto-permutable, for each $\alpha \in Aut(G)$.*

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Proof. For all $\alpha, \beta \in Aut(G)$, we have

$$H^\alpha (H^\alpha)^\beta = (H^\alpha)^\beta H^\alpha,$$
$$H^\alpha H^{\beta\alpha} = H^{\beta\alpha} H^\alpha,$$
$$H^\alpha H^{\alpha\beta} = H^{\alpha\beta} H^\alpha.$$

Now by the assumption we have $(HH^\beta)^\alpha = (H^\beta H)^\alpha$ and hence $HH^\beta = H^\beta H$. As H is self-auto-permutable, we obtain $H = H^\beta$. Therefore $H^\alpha = H^{\alpha\beta} = H^{\beta\alpha}$ and the proof is complete. □

If H is a subgroup of group G, we define

$$N^*_G(H) = \{\alpha \in Aut(G); \quad H^\alpha = H\}$$

to be the normalizer of H in $Aut(G)$. Then we have the interesting following result:

Lemma 2.5. *Let H be a self-auto-permutable subgroup of a group G, then the following statements hold:*

(i) If K is a subgroup of G, with $H \leq K \leq N_G(H)$, then $N^*_G(K) \leq N^*_G(H)$;

(ii) $N^*_G(N_G(H)) \leq N^*_G(H)$;

(iii) If K is a characteristic subgroup of G and contained in $N_G(H)$ then $N^*_G(KH) = N^*_G(H)$.

Proof. (i) For each $\alpha \in N_{Aut(G)}(K)$, we have $H^\alpha \leq K^\alpha = K \leq N_G(H)$. Therefore $HH^\alpha = H^\alpha H$ and by the assumption $H = H^\alpha$, which shows that $\alpha \in N_{Aut(G)}(H)$.
(ii) Clearly, $H \leq N_G(H) \leq N_G(H)$. Clearly, (i) implies that $N_{Aut(G)}(N_G(H)) \leq N_{Aut(G)}(H)$. Conversely, for each $\alpha \in N_{Aut(G)}(H)$ and $g \in N_G(H)$ we have $H = H^\alpha$ and $H = H^g$. They follow that

$$H = H^\alpha = (H^g)^\alpha = (H^\alpha)^{\alpha(g)} = H^{\alpha(g)}.$$

Hence $\alpha(g) \in N_G(H)$ and so $\alpha \in N_{Aut(G)}(N_G(H))$.
(iii) Clearly, $N_{Aut(G)}(H)$ is contained in $N_{Aut(G)}(KH)$. We also have $H \leq KH \leq N_G(H)$ and part (i) gives that $N_{Aut(G)}(KH) \leq N_{Aut(G)}(H)$. □

Lemma 2.6. *Let M and L be subgroups of G, and suppose that $LM = ML$,
$(\mid K \mid, \mid N \mid) = 1$ and $Aut(G) = N^*_G(M).N^*_G(L)$. Then L is self-auto-permutable in G.*

Lemma 2.7. *Let A and B be subgroups of a group G, such that $Aut(G) = Aut(A)Aut(B)$, H be a self-auto-permutable subgroup of B and fixed by the automorphisms of A. Then H is self-auto-permutable in G.*

Proof. Assume that some $\alpha \in Aut(G)$ satisfies $HH^\alpha = H^\alpha H$. then there exist $\gamma \in Aut(A)$ and $\beta \in Aut(B)$ such that $\alpha = \gamma\beta$ and $HH^{\gamma\beta} = H^{\gamma\beta}H \Rightarrow H(H^\gamma)^\beta = (H^\gamma)^\beta H$. By hypothesis, $H^c A$, so $HH^\beta = H^\beta H$. Applying the condition that H is self-auto-permutable in B, we get $H = H^\beta$. Consequently, $H = H^{\gamma\beta} = H^\alpha$ and H is self-auto-permutable in G. □

Definition 2.8. *Let G be a group. A subgroup H of G is called weakly characteristic in G if $H^\alpha \leq N_G(H)$ implies that $H^\alpha = H$, where $\alpha \in Aut(G)$.*

Definition 2.9. *Let G be a group. A subgroup H of G is said to satisfy subcharacteriser condition in G if for every subgroup K of G such that HK, it follows that $N^*_G(K) \leq N^*_G(H)$.*

Theorem 2.10. *Let H be a subgroup of G. Then:
(i) If H is self-auto-permutable in G, then H is weakly characteristic in G;
(ii) If H is weakly characteristic in G, then H satisfies the subcharacteriser condition in G.*

Theorem 2.11. *Let H be a p-subgroup of a group G. The following properties are equivalent:
(i) H is a self-auto-permutable subgroup of G;
(ii) H is a weakly characteristic subgroup of G;
(iii) H satisfies the subcharacteriser condition in G.*

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Valuation Rings and Modules

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Abstract: The purpose of this paper is to compare and investigate relations between valuation rings and valuation modules.

Keywords: multiplication module, valuation ring, valuation module.

1 INTRODUCTION

Throughout this paper, R denotes an integral domain, with quotient field $K, T = R - \{0\}$ and M is a unitary R-module. An R-module M is called a multiplication R-module, if for each submodule N of M, there exists an ideal I of R such that $N = IM$. (For more information about multiplication modules, see[2, 4]). An integral domain R is called a valuation ring, if for each $x \in K = R - \{0\}$, $x \in R$ or $x^{-1} \in R$. In [3], valuation modules in case module is torsion-free investigated. Morover in [1], nofinitely generated submodules of faithful multiplication valuation modules is investigated.

2 Valuation Rings

Definition 2.1. *A subring R of a field K is called a valuation ring of K if for every $\alpha \in K, \alpha \neq 0$, either $\alpha \in R$ or $\alpha^{-1} \in R$.*

Example 2.2. *1) Any field of K is a valuation ring of K.*

2) Let p be a fiexd prime. Let $R \subset \mathbb{Q}$, the field of rationals, be definded by $R = \{p^r \frac{m}{n} | r \geq 0, (p, m) = (p, n) = (m, n) = 1\}$. Then R is a valuation ring of $\mathbb{Q}$.

Proposition 2.3. *Let V be a valuation ring of K. Then*

1) K is the qoutiont field of V.

2) Any subring of K containing V is a valuation ring of K.

3) V is a local ring.

4) V is integrally closed.

Proposition 2.4. *The ideals of a valuation ring are totally ordered by inclusion. Conversely if the ideals of domain V with quotient field K are totally ordered by inclusion, then V is a valuation ring of K.*

Corollary 2.5. *If V is a valuation ring of K and P is a prime ideal of V, then V_p and $\frac{V}{P}$ are valuation ring.*

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Corollary 2.6. *Any Noetherian valuation ring is a principal ideal domain.*

Corollary 2.7. *Let V be a Noetherian valuation ring. Then there exists an irredusible element $p \in V$ such that every ideal of V is of the type $I = (p^m), m \geq 1$ and $\bigcap_{m=1}^{\infty}(p^m) = 0$.*

3 Valuation Modules

Let R be an integral domain with quotient field K and M a torsionfree R-module. For $y = \frac{r}{s} \in K$ and $x \in M$, we say that $yx \in M$ if there exists $m \in M$ such that $rx = sm$.

Lemma 3.1. *Let R be an integral domain with quotient field K and M a torsionfree R-module. Then the following conditions are equivalent:*

1) *For all $y \in K$ and all $x \in M$, $yx \in M$ or $y^{-1}M \subseteq M$;*

2) *For all $y \in K$, $yM \subseteq M$ or $y^{-1}M \subseteq M$.*

Definition 3.2. *Let R be an integral domain with quotient field K. A torsionfree R-module M is called valuation module (VM) if one of the condition of Lemma 3.1 holds.*

Example 3.3. 1) *Any vector space is a valuation module.*

2) *Let R be a domain. R is a valuation ring if and only if R is a valuation R-module.*

3) *Let $R = \mathbb{Z}$ and p be a prime integer number. If $M = \{p^n\frac{a}{b} | a, b, n \in \mathbb{Z}, b \neq 0, n \geq 1, (p, a) = (p, b) = (a, b) = 1\}$ then M is a valuation module.*

4) *$\mathbb{Z}$ is not a valuation $\mathbb{Z}$-module.*

an R-module M is said to be integrally closed whenever $y^n m_n + \cdots + ym_1 + m_0 = 0$ for some $n \in \mathbb{N}, y \in K$ and $m_i \in M$, then $ym_n \in M$.

Lemma 3.4. *Any valuation module is integrally closed.*

Proposition 3.5. *Let K be the quotient field of a domain R and M a torsionfree R-module. Let S be the set, ordered by inclusion, of all non-empty subsets of M. Then the following conditions are equivalent:*

1) *M is a valuation module;*

2) *$S' = \{(N : M)|N \in S\}$ is totally ordered;*

3) *For $U = \{rM|r \in R\}$ the subset of S, U' is totally ordered.*

Corollary 3.6. *Let R be a domain and M a torsionfree R-module. Then M is a valuation module if and only if for any submodules N, L of M, $(N : M) \subseteq (L : M)$ or $(L : M) \subseteq (N : M)$.*

Corollary 3.7. *Let R be a domain and M a faithful multiplication R-module. Then M is a valuation module if and only if for any two submodules N, L of M, $N \subseteq L$ or $L \subseteq N$.*

Remark 3.8. *$\mathbb{R}^2$ is a valuation R-module, but not a multiplication R-module. Note that $\mathbb{R} \oplus (0) \nsubseteq (0) \oplus \mathbb{R}$ and $(0) \oplus \mathbb{R} \nsubseteq \mathbb{R} \oplus (0)$.*

Note that R does not have non-zero maximal submodules as an R-module. Any vector space is a VM, but an infinite dimensional vector space has infinite number of maximal submodules. So it is not necessary that each valuation module has a (unique) maximal submodule.

Theorem 3.9. *Let M be a valuation R-module. Then the following statements are true.*

1) *For any submodule N of M, such that $\frac{M}{N}$ is a torsionfree R-module, $\frac{M}{N}$ is a VM.*

2) *If M is finitely generated, then for each $p \in Spec(R)$, M_p is a valuation R_p-module.*

3) *If M' is a torsionfree R-module and $\varphi : M \longrightarrow M'$ is an epimorphism, then M' is a valuation module too.*

The following give the relations between valuation rings and valuation modules.

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Lemma 3.10. *Let R be a valuation ring and M a torsionfree R-module. Then M is a valuation R-module.*

Lemma 3.11. *If M is a multiplication valuation R-module, then M is finitely generated and R is a valuation ring.*

Lemma 3.12. *Let R be a valuation domain. Then every finitely generated torsion-free R-module is free.*

Lemma 3.13. *Let R be a domain. Then R is a valuation ring if and only if every free R-module is a valuation module.*

Corollary 3.14. *Let M be a multiplication valuation module over an integral domain R. Then M is isomorphic to R.*

An element u of an R-module M is said to be unit provided that u is not contained in any maximal submodule of M. In a multiplication R-module M, $u \in M$ is unit if and only if $M = Ru$.

Theorem 3.15. *Let R be a local ring (not necessarily an integral domain) with unique principal maximal ideal $I = (p)$ and M a multiplication R-module such that $\bigcap_{n=1}^{\infty}(p^n)M = (0)$. Then the only proper submodules of M are (0) and $(p^m)M$, for some $m \geq 1$. Furthermore, if M is faithful, then either p is nilpotent or M is a valuation module.*

Theorem 3.16. *Let M be a finitely generated module over an integrally closed ring R. If M is a valuation module, then M is a free R-module and R is a valuation ring.*

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Number of undirected cayley graph of finite group Z_n and there isomorphism graphs

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Abstract: Let G be a finite group and S be a nonempty subset of G that not containing the identity element 1. The cayley graph $\Gamma = Cay(G, S)$ is simple graph who vertices are $V(\Gamma) = G$ and $E(\Gamma) = \{\{g, sg\}|g \in G, s \in S\}$. If $S = S^{-1}$ then (g , h)=(h , g) and the cayley graph is undirected. In this paper, we specify the subset S for the graph Z_n , to find number of cayley graphs that are undirected and determin how tow undirected cayley graphs are isomorphism.

Keywords: cayley graph; cayley subset; finite group; isomorphism of graph.

1 Introduction

Definition of this consepts are given later. In this section cosequence main result by the following theorems.

Theorem 1.1. *Let G be a Finite group. Number of cayley graph that are undirected and contrasting with group of Z_n are :*

a) if n=1 then nonexist the undirected cayley graph for gruop of Z_1 .

b) if n=2 or 3 then number of cayley graph that undirected is one.

c) if $n \geq 4$

i) if n is even then number of undirected cayley graph is

$$\binom{\frac{n}{2}}{1}+\binom{\frac{n}{2}}{2}+\binom{\frac{n}{2}}{3}+\cdots+\binom{\frac{n}{2}}{\frac{n}{2}-1}+\binom{\frac{n}{2}}{\frac{n}{2}} \quad (1)$$

ii) if n is odd then number of undirected cayley graph is

$$\binom{\frac{n-1}{2}}{1}+\binom{\frac{n-1}{2}}{2}+\binom{\frac{n-1}{2}}{3}+\cdots+\binom{\frac{n-1}{2}}{\frac{n-1}{2}}+\binom{\frac{n-1}{2}}{\frac{n-1}{2}} \quad (2)$$

For a finite group G , a nonempty subset S of G is called a cayley subset if $S = S^{-1} := \{s|s \in S\}$ and $1 \notin S$ .Given a cayley subset S of G , the cayley graph $\Gamma = cay(G, S)$ of G with respect to S consists of the vertex set $V\Gamma = G$ and the edge set $E\Gamma = \{\{g, sg\}|g \in G, s \in S\}$. Note that $S = S^{-1}$ thus Γ is undirected . It is not difficult to see that Γ is the disjoint union of k copies of $cay(\langle S\rangle, S)$ when $k = |G, \langle S\rangle|$. Hence Γ is connected if and only if $\langle S\rangle = G$. Let X and Y are graphs then X and Y are isomorphism if exist function f such that $f : V(X) \longrightarrow V(Y)$ and if $\{x, y\} \in E(X) \Longleftrightarrow \{f(x), f(y)\} \in E(Y)$

Theorem 1.2. *$cay(Z_n, S_1)$ and $cay(Z_n, S_2)$ are isomorphism if :*

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a) $\mid S_1 \mid=\mid S_2 \mid$
a) $S_1 = kS_2, k \in Z$

2 preliminare

We need the lemma 2.1 and lemma 2.2 to proof theorem 1.1.

Lemma 2.1. *In a cayley graph* $\Gamma = Cay(G,S)$ *, if* $|s| = k$ *then* Γ *is k-regular.*

Lemma 2.2. *For the group* Z_n *, number of cayley graphs* $\Gamma = cay(G,S)$ *that are undirected and k-regular is*

a) If n is odd and $1 \le k \le \frac{n-1}{2}$ *that k is even and*

$$\binom{\frac{n-1}{2}}{\frac{k}{2}}$$

b) If n is even and $1 \le k \le \frac{n}{2} - 1$

i) If k even

$$\binom{\frac{n}{2}-1}{\frac{k}{2}}$$

ii) If k odd

$$\binom{\frac{n}{2}}{\frac{k+1}{2}} - \binom{\frac{n}{2}-1}{\frac{k+1}{2}}$$

Proof. a) In the beginning we proof that k is even.
The cayley graph is undirected . Then $S = S^{-1}$ and n is odd then there is not element of group Z_n that its inverse itself . Hence we can not defin one S that S has a one element and S is cayley subset of undirected cayley graph if S has at least tow element or S is union of S_i has tow element and collection of S_i distinct .Hence $k(= |s|)$ is even. In the graph of Z_n the S_i $(1 \le i \le \frac{n-1}{2})$that have tow element are:

$S_1 = \{[1], [n-1]\}$

$S_2 = \{[2], [n-2]\}$

$S_3 = \{[3], [n-3]\}$

.

.

.

$S_{\frac{n-1}{2}} = \{[\frac{n-1}{2} - 1], [\frac{n-1}{2}]\}$

Other S can build an undirected cayley graph $(i.eS = S^{-1})$ consist of union of S_i that $(1 \le i \le \frac{n-1}{2})$ and number of this S $(|S| = 6)$ are:

$$\binom{\frac{n-1}{3}}{3}$$

And similarly , number of S that are cayley subset and has k element and $Cay(G,S)$ and undirected are union of three S_i that $(1 \le i \le \frac{n-1}{2})$and number of this S $(|S| = k)$ is:

$$\binom{\frac{n-1}{2}}{\frac{k}{2}}$$

b)If n is even for $S_{\frac{n}{2}} = \{[\frac{n}{2}]\}$ we have $S_{\frac{n}{2}} = S_{\frac{n}{2}}^{-1}$ and $S_{\frac{n}{2}}$ cayley subset for undirected cayley graph that is 1-regular .Also cayley subset that are tow element and$S = S^{-1}$ are:

$S_1 = \{[1], [n-1]\}$

$S_2 = \{[2], [n-2]\}$

$S_3 = \{[3], [n-3]\}$

.

.

.

$S_{\frac{n}{2}-1} = \{[\frac{n}{2} - 1], [\frac{n}{2} + 1]\}$
Union of S_i that $(1 \le i \le \frac{n}{2} - 1)$ and $S_{\frac{n}{2}}$ have k element that k is odd and any union of S_i $(1 \le i \le \frac{n}{2} - 1)$ is even number .
Now number of undirected cayley graph that building with cayley subset S $(S = S^{-1})$ and $|S|$ is odd

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equal :

$$\binom{\frac{n}{2}}{\frac{k+1}{2}} - \binom{\frac{n}{2}-1}{\frac{k+1}{2}}$$

and number of undirected k-regular cayley graph that k is even equal:

$$\binom{\frac{n}{2}-1}{\frac{k}{2}}$$

□

3 proof of theorem 1.1

a) it is obviously.
b) If $G = Z_2$ or Z_3 then number of cayley graph subset S that $(S = S^{-1})$ is single, that is $\{[1]\}$, and for group Z_3 cayley graph subset S that $(S = S^{-1})$ is $\{[1], [2]\}$
c) (i)if n be even by lemma 3(b) number of undirected cayley graph for group of Z_n is equal to number cayley subset S that $(S = S^{-1})$ is equal to sum of number of undirected cayley graph that are k-regular $(k \in \{1, 2, 3, ..., \frac{n}{2}\})$ and is equal to numbrr of cayley subset S that $|S| = k$ $(k \in \{1, 2, 3, ..., \frac{n}{2}\})$ that is :

$$1+(\frac{n}{2}-1)+\left(\binom{\frac{n}{2}}{2}-\binom{\frac{n}{2}-1}{2}\right)+\binom{\frac{n}{2}-1}{2}+\left(\binom{\frac{n}{2}}{3}-\binom{\frac{n}{2}-1}{3}\right)+\binom{\frac{n}{2}-1}{3}$$

$$+\left(\binom{\frac{n}{2}}{4}-\binom{\frac{n}{2}-1}{4}\right)+...+\left(\binom{\frac{n}{2}}{\frac{n}{2}-1}-\binom{\frac{n}{2}-1}{\frac{n}{2}-1}\right)+\binom{\frac{n}{2}-1}{\frac{n}{2}-1}+\binom{\frac{n}{2}}{\frac{n}{2}}$$

$$= \frac{n}{2} + \binom{\frac{n}{2}}{2} + \binom{\frac{n}{2}}{3} + \cdots + \binom{\frac{n}{2}}{\frac{n}{2}-1} + \binom{\frac{n}{2}}{\frac{n}{2}}$$

$$= \binom{\frac{n}{2}}{1} + \binom{\frac{n}{2}}{2} + \binom{\frac{n}{2}}{3} + \cdots + \binom{\frac{n}{2}}{\frac{n}{2}-1} + \binom{\frac{n}{2}}{\frac{n}{2}}$$

ii) If n be odd by lemma 3(a) number of undirected cayley graph for group of Z_n is :
number of undirected cayley graph that are 2-regular
+ number of undirected cayley graph that are 4-regular
+ number of undirected cayley graph that are 6-regular
.
.
.
+ number of undirected cayley graph that are $(n-1)$-regular

$$= \binom{\frac{n-1}{2}}{1} + \binom{\frac{n-1}{2}}{2} + \binom{\frac{n-1}{2}}{3} + \cdots + \binom{\frac{n-1}{2}}{\frac{n-1}{2}-1} + \binom{\frac{n-1}{2}}{\frac{n-1}{2}}$$

4 proof of theorem 1.2

if $| S_1 |=| S_2 |$ and $S_2 = kS_1, k \in Z$ then $f : Z_n \longrightarrow Z_n$ that $f(a) = ka$ is isomorphism because:

$\forall a, b \in Z_n$
$if a \sim b \Rightarrow ab^{-1} \in S_1$
$\Rightarrow a - b \in S_1$
$\Rightarrow k(a - b) \in kS_1$
$\Rightarrow ka - kb \in kS_1$
$\Rightarrow (ka)(kb)^{-1} \in S_2$
$\Rightarrow ka \sim kb$
$\Rightarrow cay(Z_n, S_1) \cong cay(Z_n, S_2)$

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استفاده از رمزگذاری فازی اثر انگشت در تشخیص هویت

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چکیده: سیستم های بیومتریک سیستم هایی هستند که قادرند نمونه تشخیص هویت را استخراج کرده و داده مرجع را مقایسه و همسانی هویت ادعا شده توسط فرد را مشخص کنند. جهت تشخیص هویت به شناسه هایی از جمله: انگشتان، تصاویر صورت، تایپ کردن و راه رفتن توجه می کنند. در سیستم های تشخیص هویت با استفاده از بیومتریک دو ویژگی اصلی وجود دارد که یکی نرخ پذیرش غلط (FAR) و دیگری نرخ رد غلط (FRR) است. ما در این مقاله روش منطق فازی در سیستم های اثر انگشت، معماری یک سیستم ترکیبی و استفاده از رمز گذاری فازی را برای نشان دادن ساختار ویژگی های برآمدگی های اثر انگشت مطرح می کنیم.
کلمات کلیدی: منطق فازی، بیومتریک، اثرانگشت

مقدمه

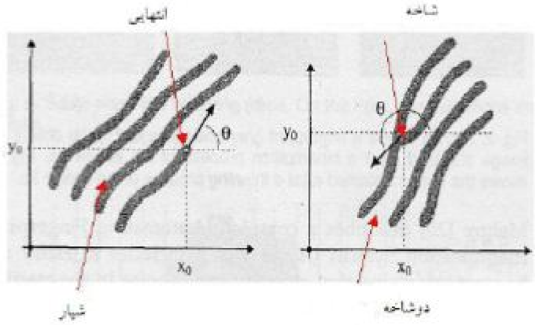
در سال۱۹۵۶ کشف شد که اثر انگشت انسان منحصر به فرد است . با این روش توانستند از جنایت کارانی که اثر انگشتشان در صحنه جرم وجود داشت تعیین هویت کنند. با وجود روش های مبتکرانه بهره وری ازسیستم های تشخیص هویت دستی، زمانی که حجم کار زیاد می شد نمی توانستد تمام خواسته ها را انجام دهند. با رشد سریع تکنولوژی شاهد آن هستیم که ارتباط بین افراد بیش از پیش الکترونیکی شده است لذا نیاز داریم تا هویت افراد را بطور دقیق و خودکار تعیین کنیم. اطلاعات بیومتریک هر فرد مخصوص همان فرد است و امکان فراموش کردن و گم شدن آن توسط فرد وجود ندارد و دیگران نمی توانند به آن دسترسی داشته باشند. لذا سیستم های بیومتریک نسبت به سیستم های سنتی قابلیت اطمینان بالاتری دارند.چون کنترل فازی به یک مدل ریاضی نیاز ندارد، می توان در مورد خیلی از سیستم هایی که به وسیله ی تئوری کنترل متعارف قابل پیاده سازی نیستند آن را به کار برد.

بیومتریک

روش هاي تأييد هويت به سه فاكتور تقسيم بندي مي شوند: الف) اطلاعاتی كه كاربران مي دانند مانند رمز عبور. ب) اطلاعاتی كه كاربران به همراه دارند مانند كارتهاي خود پرداز، كارت هاي هوشمند. ج)اطلاعاتی كه مربوط به خود كاربران است مانند بيومتريک ها شامل اثر انگشت، الگوی شبكيه و عنبيه.

دسته سوم (بیومتریک ها) امن ترین و ساده ترین فاکتور تأیید هویت هستند. بیومتریک به روش های خودکار تشخیص یا تایید هویت یک شخص زنده از طریق اندازه گیری مشخصه های فیزیولوژیکی یا رفتاری وی اطلاق می شود. یک اثر انگشت از برآمدگی ها و شیارهای متعددی تشکیل شده است. این برآمدگی ها و شیارها تشابهاتی دارند، چون موازی بوده و پهنای معادل دارند . اما براساس تحقیقات وسیعی که در امرتشخیص اثر انگشت انجام شده، این نتیجه به دست آمده که شیارها وبرآمدگی ها هیچ تاثیری در تشخیص ندارند. اثر انگشت را نمی توان با تصاویر مقایسه کرد. نقاط غیرطبیعی در شیارها را منوشیا می نامیم. درمیان انواع متفاوتی از منوشیاها دو مورد بسیار مهم

وجود دارد . یکی انتهایی که سر انتهایی یک شیار است و دیگری دو شاخه نامیده می شود که نقطه ای در شیار بوده و آن نقطه را به دو شیار تقسیم می کند.



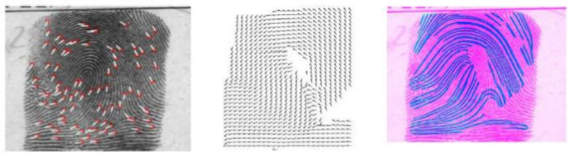
هر نقطه منوشیا که چند برآمدگی را با هم پیوند داده است دو مختصات (X, Y) و یک اندازه گیری برای کیفیت اثر انگشت دارد. تطبیق اثر انگشت بستگی به محل و چرخش روی آن دارد . گروهی از نقاط مختصاتشان با هم هماهنگ نیست لذا هراثرانگشت نمی تواند نمایش داده شود. در بسیاری از موارد احتمال در تعلق یک اثر انگشت به فرد، زیر مجموعه ای بین ۰ و ۱ می باشد و این باعث می شود که ما برای محاسبه آن از منطق فازی استفاده کنیم.

منطق فازی در سیستم های بیومتریک

برای سطح پذیرش های نادرست و خطا هایی که در عدم پذیرش نادرست ایجاد می شود پارامتری بنام آستانه را به کار می بریم. تصمیم گیری که سیستم برای شناسایی یک فرد انجام می دهد براساس مقدار نزدیک به آستانه (T) می باشد. زمانی که مقدار آستانه افزایش پیدا می کند، احتمال پذیرش نادرست(کاربر غیر مجاز) کاهش پیدا می کند و احتمال رد اشتباه افزایش می یابد.

رمزگذاری فازی برای نمایش ساختار برآمدگی ها

وجود نویزها در تصاویر با کیفیت پایین باعث می شود که خطاهای زیادی در هنگام استخراج منوشیاها رخ دهد. از آنجا که الگوهای اثر انگشت در طبیعت مبهم هستند و برآمدگی های پایانی توسط زخم ها به آسانی تغییر می کنند از برآمدگی های دوشاخه به عنوان منوشیاهای اثر انگشت استفاده کرده ایم. همچنین یک ویژگی تصویر فازی رمزگزار را با استفاده از توابع عضویت مخروطی شکل برای نشان دادن ساختار ویژگی برآمدگی های دو شاخه استخراج شده از اثر انگشت طراحی کرده ایم.



این رمزگزاری شامل سه مرحله می باشد. ابتدا هر تصویر اثر انگشت را که سایز آن ۵۱۲× ۵۱۲ است به بخش های ۸× ۸ با پهنای ۶۴ پیکسل تقسیم می کنیم .در مرحله بعد یک مقدار عضویتی برای هر دوشاخه اثر انگشت در یک تابع مخروطی شکل برای هر بخش اثرانگشت به منظور نشان دادن ساختاری از دوشاخه ها در نظر می گیریم. نتایج حاصل از این تجزیه برای به دست آوردن مقدار عضویت دوشاخه ها در مجموعه های فازی استفاده می شود.

نتایج

توابع عضویت در بخش های اثر انگشت به صورت زیر محاسبه می شود

$$\mu(i, j) = \sum_{n=۱}^{m} (۱ - \frac{d}{w}).$$

در این فرمول $\mu(i, j)$ معرف تابع عضویت است. m معرف نقطه دو شاخه نزدیک به مرکز (i, j)، d فاصله تا مرکز شبکه و w عرض شبکه و پهنای هر بخش نیز ۶۴ پیکسل است. در مرحله ی سوم مجموعه مقادیرعضویت در هر بخش محاسبه می شود. سپس ساختار دوشاخه های تصویر فازی به دست می آید. نتایج و مراحل اصلی شناسایی اثر انگشت بر اساس رمزگذاری فازی بدین شرح است که ابتدا با کاهش نویز های تصویر خصوصیات مهم اثر انگشت را مشخص می کنیم و به نرمال سازی کاهش عمق و سطح خطوطی که توسط اختلاف فشار اثر انگشت ایجاد شده اند می پردازیم. به کمک فیلتر گابور نویز های نامطلوب را حذف و برآمدگی ها و فرو رفتگی های درست را حفظ

می کنیم. بعد از رقیق کاری تصویر با استفاده از الگوی بیومتریک از اثر انگشت، منوشیا های اثر انگشت را استخراج می کنیم.

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Autocommutativity degree of an elementary abelian p-group

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Abstract: In this paper we define autocommutativity degrees of finite groups. If G is a finite group, the autocommutativity degree of G, denoted by $d_{aut}(G)$, is the probability that an element of G is fixed by an automorphism of G. We also determine an upper bound for autocommutativity degree of elementary abelian p-groups.
Keywords: Autocommutativity degree, Autoabelian, Absolute centre..

1 INTRODUCTION

If G is a finite group then the commutativity degree of G, denoted by

$$d(G) = \frac{1}{|G|^2} \left| \{(x,y) \in G \times G | [x,y] = 1\} \right|,$$

is the probability that two randomly chosen elements of G commute. The commutative degree first studied by Gustafson in 1973, where he showed that $d(G) \leq \frac{5}{8}$ for every non-abelian finite group G. In 1995 Lescot investigated this concept by considering the notion of commutativity degree of finite groups. Whence he obtained certain results in this regard. In this paper, we introduce the *autocommutativity degree*, denoted by $d_{aut}(G)$, which are defined as follows:

$$d_{aut}(G) = \frac{|\{(x,\alpha) \in G \times Aut(G) | [x,\alpha] = 1\}|}{|G||Aut(G)|},$$

and where $[x,\alpha] = x^{-1}\alpha(x)$. In Hegarty [?], the characteristic subgroups $K(G)$ and $L(G)$ of G are defined as follows:

$$K(G) = \langle\ [x,\alpha] \mid x \in G, \alpha \in Aut(G)\ \rangle,$$

and

$$L(G) = \{\ x \mid [x,\alpha] = 1\ \forall \alpha \in Aut(G)\ \},$$

which are called autocommutator subgroup and absolute centre of G, respectively. One can easily check that $K(G)$ contains the derived subgruop G' of G and $L(G)$ is contained in the centre, $Z(G)$, of G.

Definition 1.1. *A group G is said to be autoabelian if $K(G) =< 1 >$ or $L(G) = G$.*

Obviously, G is autoabelian if and only if $d_{aut}(G) = 1$.

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2 RESULTS ON THE AUTO-COMMUTATIVITY DEGREE

Let G be a group and α be an automorphism of G. The subgroup $C_G(\alpha)$ of G is defined by

$$C_G(\alpha) = \{ \ x \in G \mid [x, \alpha] = 1 \ \ \}.$$

The following theorem gives the upper bound for $d_{aut}(G)$ when G is elementary abelian, which is similar to the Lemma 1.3 of [**?**].

Theorem 2.1. *Let G be a non-autoabelian p-group and elementary abelian of rank n $(n \geq 2)$. Then*

$$d_{aut}(G) \leq \frac{1}{p^n(p^n-1)...(p^n-p^{n-2})} + \frac{1}{p}.$$

Proof. Let G be an elementary abelian group, then $|Aut(G)| = (p^n-1)(p^n-p)...(p^n-p^{n-1})$ and we have $|C_G(\alpha)| \leq p^{n-1}$ for $\alpha \neq 1$ which $\alpha \in Aut(G)$. Put $k = p^n(p^n-1)...(p^n-p^{n-1})$, hence

$$\begin{aligned}
kd_{aut}(G) &= p^n(p^n-1)...(p^n-p^{n-1})d_{aut}(G) \\
&= |G||Aut(G)|d_{aut}(G) \\
&= \textstyle\sum_{\alpha \in Aut(G)} |C_G(\alpha)| \\
&= \textstyle\sum_{\alpha = \{1\}} |C_G(\alpha)| + \sum_{\alpha \in Aut(G) \setminus \{1\}} |C_G(\alpha)| \\
&= |G| + |C_G(\alpha)|(|Aut(G)|-1) \\
&\leq p^n + p^{n-1}[(p^n-1)...(p^n-p^{n-1})-1] \\
&= p^{n-1}(p-1) + p^{n-1}(p^n-1)...(p^n-p^{n-1}).
\end{aligned}$$

Therefore

$$d_{aut}(G) \leq \frac{1}{p^n(p^n-1)...(p^n-p^{n-2})} + \frac{1}{p}.$$

□

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A Characterization of Unique τ-Coclosure Modules

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Abstract: We give a characterization of unique coclosure modules with respect to a cohereditary torsion theory τ.

Keywords: τ-Coclosed modules; τ-UCC modules; Cohereditary torsion.

1 INTRODUCTION

Throughout this paper R will denote an arbitrary associative ring with identity and M will be unitary right R-module.The notation $N \leq^{\oplus} M$ denotes that N is a direct summand in M; $N \ll M$ means that N is small in M (i.e. for all proper submodule L of M, $L + N \neq M$). A module N is said to be *small* if $N \ll L$, for some module L. For $N, L \leq M, N$ is *supplement* of L in M if $N + L = M$ with $N \cap L \ll N$. Following [4], a module M is called supplemented if every submodule of M has a supplement in M. On the other hand, the module M is *amply supplemented* if, for any submodules A, B of M with $M = A + B$ there exists a supplement P of A in M such that $P \leq B$. Module M is called a *weakly supplemented* module if for each submodule A of M there exists a submodule B of M such that $M = A + B$ and $A \cap B \ll M$.

Let $\tau = (\mathcal{T}, \mathcal{F})$ denotes a cohereditary torsion theory on R-Mod, where $\mathcal{T}$ and $\mathcal{F}$ denote the classes of all τ-torsion and τ-torsionfree modules respectively.

If $\tau(M)$ denotes the sum of the τ-torsion submodules of M, then $\tau(M)$ is necessarily the unique largest τ-torsion submodule of M and $\tau(M/\tau(M)) = 0$ for an R-module M. Let N be a submodule of a module M. Then N is called *τ-small* in M if it is $N \in \mathcal{F}$ and small in M. In this case we write $N \ll_\tau M$. Let $B \leq A \leq M$, if $A/B \ll_\tau M/B$, then B is called a *τ-cosmall* submodule of A in M (denoted by $B \overset{\tau\text{-}cs}{\hookrightarrow} A$). A submodule A of M is called *τ-coclosed* (denoted by $A \overset{\tau\text{-}cc}{\hookrightarrow} M$), if A has no proper τ-cosmall submodule. A *τ-coclosure* of a submodule B of M is a τ-cosmall submodule of B in M which is also a τ-coclosed submodule of M. A module M is called a *unique τ-coclosure module* (denoted by *τ-UCC* module) if every submodule of M has a unique τ-coclosure in M.

In [2] Doğruöz, Harmanci, and Smith studied modules in which every submodule has a unique closure with respect to a hereditary torsion theory τ. In this paper we investigate modules in which every submodule has a unique coclosure with respect to a cohereditary torsion theory τ.

2 Main Results

Theorem 2.1. *Let R be a ring and let τ be a cohereditary torsion theory on Mod-R. If M is an amply τ-supplemented R-module, then the follow-*

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ing statements are equivalent.

(1) *M is a* $\tau - UCC$ *module;*

(2) *for all sets* I *and for all submodules* $K_i \overset{\tau\text{-}cs}{\hookrightarrow} L_i$ *in* M *for all* $i \in I$, $\bigcap_{i\in I} K_i \overset{\tau\text{-}cs}{\hookrightarrow} \bigcap_{i\in I} L_i$ *in* M.

(3) *if* $K \overset{\tau\text{-}cs}{\hookrightarrow} K'$ *in* M *and* $L \overset{\tau\text{-}cs}{\hookrightarrow} L'$ *in* M, *then* $(K \cap L) \overset{\tau\text{-}cs}{\hookrightarrow} (K' \cap L')$.

(4) *if* $K\theta_\tau K'$ *and* $L\theta_\tau L'$, *for submodules* K, K', L *and* L' *of* M, *then* $(K \cap L)\theta_\tau(K' \cap L')$.

(5) *if* $L \overset{\tau\text{-}cs}{\hookrightarrow} K + L$ *then* $(K \cap L) \overset{\tau\text{-}cs}{\hookrightarrow} K$.

(6) *for any* $A \subseteq B \subseteq M$ *and a* τ*-coclosure* B_1 *of* B *in* M, *there exists a* τ*-coclosure* A_1 *of* A *in* M *such that* $A_1 \subseteq B_1$.

(7) *the sum of any family of* τ*-coclosed submodule of* M *is* τ*-coclosed;*

(8) *the sum of two* τ*-coclosed submodule of* M *is* τ*-coclosed.*

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Absolutely weakly quasi annihilator-flat monoids

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Abstract: In this paper we define the notions of weakly quasi annihilator-flat and right(left) absolutely weakly quasi annihilator-flat. Some of their properties are investigated. We obtain some conditions that completely 0-simple semigroups are right absolutely weakly quasi annihilator-flat.

1 INTRODUCTION

In [1] Bulman-Fleming gave annihilator-flat notion and studied its property. The present paper offer a new notion that although seems to be similar to annihilator-flat against annihilator-flat it is a strong notion that is called weakly quasi annihilator-flat. Besides its properties are investigated. In fact weakly flat implies weakly quasi annihilator-flat and weakly quasi annihilator-flat implies principally weakly flat but the converse is not necessarily true. We show that annihilator-flat doesn't imply weakly quasi annihilator-flat and vice versa. Also right(left) absolutely weakly quasi annihilator-flat is definable naturally. We show that on what conditions completely 0-simple semigroups and full transformation monoids are right absolutely weakly quasi annihilator-flat. In the following we utter some definitions that are used in this paper.
Let S be a semigroup. An element $s \in S$ is called *left cancellable* (resp. *right cancellable* if $rs = ts$) if $sr = st$ implies $r = t$, for all $r, t \in S$. Moreover, the semigroup S is called *cancellative* if all elements of S are left and right cancellable.

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Let S be a monoid. A non-empty set A is called a *right S-act* usually denoted A_S, if S acts on A unitarily from the right that is there exists a mapping $A \times S \to A$, $(a, s) \mapsto as$, satisfying the condition $(as)t = a(st)$ and $a1 = a$, for all $a \in A$ and all $s, t \in S$. Left S-act ${}_SA$ is defined dually. Let A_S be a right S-act. A is called *flat* if the functor $A\otimes-$ preserves embeddings of left S-acts. If this functor preserves embeddings of (principal) left ideals of S into S, then A is called *(principally) weakly flat.*
An act A_S is called *torsion free*, if $as = bs$ with $a, b \in A$ and s a right cancellable element of S, implies $a = b$. A left ideal J of a monoid S is called *right-stabilizing* if $j \in jJ$ for every $j \in J$(see p. 230 of [3], where the left-right dual of this property is called *Condition (LU)*). A left S-act ${}_SB$ is called *connected* whenever $b, b' \in B$ then there exist elements $b_i \in B$ and $s_i, t_i \in S$ such that

$$\begin{aligned} b &= s_1b_1 \\ t_1b_1 &= s_2b_2 \\ &\cdots \\ t_{n-1}b_{n-1} &= s_nb_n \\ t_nb_n &= b'. \end{aligned}$$

Let $a, a' \in A$ and $b, b' \in B$. Then $a \otimes b = a' \otimes b'$ in $A_S \otimes {}_S B$ if and only if there exist $s_1, ..., s_k, t_1, ..., t_k \in S$, $a_1, ..., a_{k-1} \in A_S$ and $b_1, ..., b_k \in {}_S B$ such that

$$\begin{array}{rclrcl} & & & s_1 b_1 & = & b \\ a s_1 & = & a_1 t_1 & s_2 b_2 & = & t_1 b_1 \\ a_1 s_2 & = & a_2 t_2 & s_3 b_3 & = & t_2 b_2 \\ & \cdots & & & \cdots & \\ a_{k-1} s_k & = & a' t_k & b' & = & t_k b_k. \end{array}$$

2 Right Absolutely Weakly Quasi Annihilator-Flat Monoids

In this section we define the notions of weakly quasi annihilator-flat and right(left) absolutely weakly quasi annihilator-flat and investigate some of their properties.

Definition 2.1. *Let S be a monoid and $s, t \in S$. The set ${L^*}_{s,t} = \{u \in S| \;\; us = vt, \text{ for some } v \in S\}$ (resp. ${R^*}_{s,t} = \{u \in S| \;\; su = tv, \text{ for some } v \in S\}$) is called a left(right) quasi annihilator ideal of S if it is a non-empty set.*

The following example shows that the equality $L^*_{s,t} = L^*_{t,s}$ is not generally hold.

Example 2.2. *Let $S = \{0, x, 1\}$ be a monoid, where $x^2 = 0$. Then we have $S = L^*_{0,x} \neq L^*_{x,0} = \{0, x\}$.*

Definition 2.3. *An act A_S is called weakly quasi annihilator-flat if the natural mapping $A \otimes {L^*}_{s,t} \to A$, $a \otimes u \mapsto au$ is injective, for all $s, t \in S$.*
Every weakly flat right act is obviously weakly quasi annihilator-flat, and by the fact that ${L^*}_{1,t} = St$, for all $t \in S$ it is clear that every weakly quasi annihilator-flat right act is principally weakly flat. Therefore we have

Flat ⇒ Weakly Flat ⇒ **Weakly Quasi annihilator-Flat** *⇒ Principally Weakly Flat ⇒ Torsion Free.*

Definition 2.4. *A monoid is called right(left) absolutely weakly quasi annihilator-flat if all of its right(left) acts are weakly quasi annihilator-flat.*

Theorem 2.5. *A monoid S is right absolutely weakly quasi annihilator-flat if and only if, for all $u, v, s, t \in S$ if u and v belong to ${L^*}_{s,t}$ then there exist $e_1, ..., e_{2m+1} \in L^*_{s,t}$ such that*

$$\begin{array}{rcl} u & = & ue_1 \\ ve_1 & = & ve_2 \\ ue_2 & = & ue_3 \\ & \cdots & \\ ue_{2m} & = & ue_{2m+1} \\ ve_{2m+1} & = & v. \end{array}$$

Corollary 2.6. *It is clear that if S is a right absolutely weakly quasi annihilator-flat then every left quasi annihilator ideal ${L^*}_{s,t}$ of S is right stabilizing.*

Corollary 2.7. *Suppose that S is right absolutely weakly quasi annihilator-flat. Then*

(i) each proper left Rees factor act of the form $S/{L^*}_{s,t}$ is principally weakly flat;

(ii) each amalgamated coproduct $S \amalg_{{L^*}_{s,t}} S$, where $s \neq t$ is a flat left S-act.

Theorem 2.8. *Let S be a monoid such that $xyx = x^2 y$, for all $x, y \in S$. Then S is right absolutely weakly quasi annihilator-flat if and only if, for all $s, t \in S$ the set ${L^*}_{s,t}$ is non-empty and right stabilizing left ideal.*

Lemma 2.9. *If S is a right reversible monoid then for every $s, t \in S$, ${L^*}_{s,t}$ is non-empty and connected left act.*

Theorem 2.10. *The one-element right S-act Θ_S is weakly quasi annihilator-flat if and only if, each non-empty quasi annihilator ideal ${L^*}_{s,t}$ is connected left S-act.*

Theorem 2.11. *Let S be a monoid and $R^*_{s,t}$ be a proper right quasi annihilator ideal of S. Then the right Rees factor S-act $S/R^*_{s,t}$ is weakly quasi annihilator-flat if and only if each non-empty left ideal $L^*_{u,v}$ is left connected S-act and $R^*_{s,t}$ satisfies Condition (LU).*

Theorem 2.12. *All principally weakly flat right Rees factor acts of a monoid S of the form $S/R^*_{s,t}$ are weakly quasi annihilator-flat if and only if each non-empty ideal $L^*_{u,v}$ is connected left S-act.*

This section is ended with some examples. For simplicity we use the following abbreviations: Weakly flatness = WF, principally weakly flatness = PWF, weakly quasi annihilator-flatness = *WQ*-F, right absolutely annihilator-flatness = (r.a.a-f), right absolutely weakly quasi annihilator-flatness = (r.a.w.q-f).

Example 2.13.

(i). (WQ-F $\nRightarrow$ WF). Let S be a right zero semigroup and $s, t \in S^1 (s \neq t)$. Then $L^*{}_{s,t} = \emptyset$ and so by Lemma 2.9, S^1 is not right reversible. Therefore Θ_S is not weakly flat. Moreover, according to the Theorem 2.10 Θ_S, is weakly quasi annihilator-flat.

(ii). (PWF $\nRightarrow$ WQ-F). Let S be a monoid as follows:

Then $L^*{}_{g,e} = \{e, f\}$. Suppose that $L^*{}_{g,e}$ is connected. Then there exist $s_1, ..., s_n, t_1, ..., t_n \in S$, $e_1, ..., e_n \in L^*{}_{g,e}$ such that

$$\begin{aligned} e &= s_1 e_1 \\ t_1 e_1 &= s_2 e_2 \\ &\cdots \\ t_{n-1} e_{n-1} &= s_n e_n \\ t_n e_n &= f. \end{aligned}$$

Therefore according to the table and $e = s_1 e_1$ we have $e_1 = e$. Also $e = t_1 e_1 = s_2 e_2$ therefore $e_2 = e$. By following this process we conclude that $e_n = e$. Hence $e = t_n e_n = f$. So this is a contradiction and $L^*{}_{g,e}$ is not connected. Thus Θ_S is not weakly quasi annihilator-flat by Theorem 2.10, but clearly it is principally weakly flat.

(iii). If S is any right zero semigroup, then S^1 is right absolutely weakly quasi annihilator-flat.

(iv). Right normal bands(with 1 adjoined) need not be right absolutely weakly quasi annihilator-flat. If S^1 has the multiplication table then $L^*{}_{g,e} = \{e, f\}$.

(v). (Annihilator-flat $\nRightarrow$ WQ-F). According to the [1] the cyclic act $\mathbb{N}/\rho(2,4)$, where $\rho(2,4)$ is the smalest congruence on $\mathbb{N}$ identifying 2 and 4, is annihilator-flat but not torsion free.
If $\mathbb{N}/\rho(2,4)$ be weakly quasi annihilator-flat then it is torsion free and this is a contradiction.

(vi). r.a.a-f $\nRightarrow$ r.a.w.q-f. Because every left cancelative monoid is right absolutely annihilator-flat by [1], $\mathbb{N}$ has this property too. But according previous example $\mathbb{N}$ is not right absolutely weakly quasi annihilator-flat.

(vii). r.a.w.q-f $\nRightarrow$ r.a.a-f. Let S be a monoid as follows:
Then S is not right absolutely annihilator-flat but it is right absolutely weakly quasi annihilator-flat by Theorem 2.5.

(viii). (WQ-F $\nRightarrow$ annihilator-flat). It is obvious by previous example.

(ix). The null monoid $S = \{0, x, 1\}$, where $x^2 = 0$ is not right absolutely weakly quasi annihilator-flat. Indeed, $x \in L^*{}_{x,0} = \{0, x\}$ but there is no element y belong to $L^*{}_{x,0}$ such that $xy = x$. Therefore S is not right absolutely weakly quasi annihilator-flat by Theorem 2.5.

(x). The right regular monoid with table is right absolutely weakly quasi annihilator-flat by Theorem 2.5.

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3 The completely 0-simple semigroups that are right absolutely weakly quasi annihilator-flat

A semigroup S without identity will be termed right absolutely weakly quasi annihilator-flat or right absolutely annihilator-flat when the monoid S^1 has the corresponding property. Recall that a regular Rees matrix semigroup with 0 having sandwich matrix P is right absolutely annihilator-flat if and only if, for all $\mu \neq \nu \in \Lambda$, if no entry of row μ or of row ν of P is 0, then ${p_{\mu j}}^{-1} p_{\mu i} \neq {p_{\nu j}}^{-1} p_{\nu i}$ for all $i \neq j \in I$; and no two rows of P having an entry 0 have equal support. The above results appear in [1]. If P is a $\Lambda \times I$ matrix over a group with 0, the *support* of row α is the set of those $i \in I$ for which $p_{\alpha i} \neq 0$.
We will show that if every completely 0-simple semigroup be right absolutely weakly quasi annihilator-flat then it is right absolutely annihilator-flat.

Theorem 3.1. *Let $M_0(G; I, \Lambda; P)$ be a completely 0-simple semigroup, represented as a regular Rees matrix semigroup with 0. Then the monoid $S = (M_0(G; I, \Lambda; P))^1$ is right absolutely weakly quasi annihilator-flat if and only if the following conditions hold*

(i) for all $\mu \neq \nu \in \Lambda$, $a, b, a', b' \in G$, $i, j \in I$, $\mu', \nu' \in \Lambda$, if ${p_{\mu' j}}^{-1}(a')^{-1} a p_{\mu i} = {p_{\nu' j}}^{-1}(b')^{-1} b p_{\nu i}$, some entry of row μ or some entry of row ν of P are 0;

(ii) no two rows of P that at least one of them has an entry 0 have equal support.

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A categorical approach to soft S-acts

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Abstract: The purpose of this talk is to study certain categorical properties of the categories **SoftAct** of all soft S-acts and soft homomorphisms, and **WSoftAct** of all soft S-acts and weak soft homomorphisms. It is investigated the interrelations of some particular morphisms, limits and colimits in **SoftAct** and **WSoftAct** with their corresponding notions in the categories **Act-**S of all S-acts and S-maps, and **Set** of all sets and maps.
Keywords: soft S-act, soft (co)product, soft (co)equalizer, soft pullback, soft pushout

1 INTRODUCTION

The theory of soft sets was initiated by Molodtsov [3] as a new mathematical tool for dealing with uncertainties which is free from the difficulties affecting classical methods. The study on connections between soft sets and algebraic systems has been of interest for some authors. In [3], Ali et al. introduced the concept of soft S-acts for a semigroup S to characterize general fuzzy soft S-acts.

In this talk our aim is to study the interrelations of some category-theoretic concepts including particular morphisms, limits, and colimits such as soft products, soft coproducts, soft equalizers, soft coequalizers, soft pullbacks, and soft pushouts of soft S-acts with their corresponding concepts in the categories **Act-**S of all S-acts and S-maps, and **Set** of all sets and maps. For more information on some categorical ingredients of **Act-**S, see [2].

Let $\mathcal{A}$ be an S-act and $(F, A)_{\mathcal{A}}$ be a soft set over $\mathcal{A}$. Then $(F, A)_{\mathcal{A}}$, or simply (F, A), is called a *soft S-act* or *soft S-set* over $\mathcal{A}$ if $F(a) \neq \emptyset$ is a subact of $\mathcal{A}$ for all $a \in A$. The reader is referred to [1] to see some examples of soft S-acts.

Let (F, A) and (G, B) be two soft S-acts over S-acts $\mathcal{A}$ and $\mathcal{B}$, respectively. Let $f : \mathcal{A} \to \mathcal{B}$ be an S-homomorphism and $g : A \to B$ be a map. Then (f, g) is called a *soft homomorphism* from $(F, A)_{\mathcal{A}}$ to $(G, B)_{\mathcal{B}}$ if $f(F(a)) = G(g(a))$ for all $a \in A$. Also if $f(F(a)) \subseteq G(g(a))$ for all $a \in A$, then we say that (f, g) is a *weak soft homomorphism*. All soft S-acts together with all soft homomorphisms between them forms a category which is denoted by **SoftAct**. This category is a subcategory of **WSoftAct** consisting of all soft S-acts together with all weak soft homomorphisms between them.

Let P be a category-theoretic notion in **SoftAct**. We use the term "soft corresponding notion" for P. For example, by a soft monomorphism we mean a monomorphism in **SoftAct**, a soft pullback refers to a pullback in **SoftAct**, etc. Analogously, we use "w-soft corresponding notion" for a

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category-theoretic notion in **WSoftAct**.

2 MAIN RESULTS

We study some basic category-theoretic properties of **SoftAct** and **WSoftAct**. We will see that the categorical properties in **SoftAct** and **WSoftAct** are mostly dependent to their corresponding properties in **Act-**S and **Set**.

2.1 ON PARTICULAR MORPHISMS

First we study soft monomorphisms and soft epimorphisms in **SoftAct**. Then we characterize all soft isomorphisms in terms of all isomorphisms in **Act-**S and **Set**.

Theorem 2.1. *Let* $(f, g) : (F, A)_{\mathcal{A}} \to (G, B)_{\mathcal{B}}$ *be a soft homomorphism in* **SoftAct** *such that* f *and* g *be monomorphisms (epimorphisms) in* **Act-**S *and* **Set**, *respectively. Then* (f, g) *is a soft monomorphism (soft epimorphism).*

Corollary 2.2. *Let* $(f, g) : (F, A)_{\mathcal{A}} \to (G, B)_{\mathcal{B}}$ *be a soft homomorphism in* **SoftAct** *such that* f *and* g *be bimorphisms in* **Act-**S *and* **Set**, *respectively. Then* (f, g) *is a soft bimorphism.*

The following result characterizes all soft isomorphisms in terms of all isomorphisms in **Act-**S and **Set**.

Theorem 2.3. *Let* $(f, g) : (F, A)_{\mathcal{A}} \to (G, B)_{\mathcal{B}}$ *be a soft homomorphism. Then* (f, g) *is a soft isomorphism in* **SoftAct** *if and only if* f *and* g *are isomorphisms in* **Act-**S *and* **Set**, *respectively.*

2.2 ON SOFT PRODUCTS AND SOFT COPRODUCTS

In the following soft products and soft coproducts in the categories **SoftAct** and **WSoftAct** are investigated.

Problem 2.4. *The category* **SoftAct** *has a terminal object but not an initial object. The same situation holds in* **WSoftAct**.

The following result presents a connection between w-soft products in **WSoftAct** and the products in **Act-**S and **Set**.

Theorem 2.5. *Let* $\{(F_i, A_i)_{\mathcal{A}_i}\}_{i\in I}$ *be a non-empty family of soft* S*-acts. Then the soft* S*-act* $(\prod_{i\in I} F_i, \prod_{i\in I} A_i)_{\prod_{i\in I}\mathcal{A}_i}$ *is a w-soft product* $\overset{w}{\prod_{i\in I}} (F_i, A_i)_{\mathcal{A}_i}$ *of this family, where* $\prod_{i\in I} F_i : \prod_{i\in I} A_i \to \mathrm{P}(\prod_{i\in I} \mathcal{A}_i)$ *is given by* $(\prod_{i\in I} F_i)((a_i)_{i\in I}) = \prod_{i\in I}(F_i(a_i))$, $a_i \in A_i, i \in I$.

In the following, a dual result of Theorem 2.5 for soft coproducts in **SoftAct** is obtained.

Theorem 2.6. *Let* $\{(F_i, A_i)_{\mathcal{A}_i}\}_{i\in I}$ *be a non-empty family of soft* S*-acts. Then the soft* S*-act* $(\coprod_{i\in I} F_i, \coprod_{i\in I} A_i)_{\coprod_{i\in I}\mathcal{A}_i}$ *is a soft coproduct* $\coprod_{i\in I} (F_i, A_i)_{\mathcal{A}_i}$ *of this family, where* $\coprod_{i\in I} F_i : \coprod_{i\in I} A_i \to \mathrm{P}(\coprod_{i\in I} \mathcal{A}_i)$ *is given by* $(\coprod_{i\in I} F_i)(a_i, i) = F_i(a_i) \times \{i\}$, *for all* $a_i \in A_i, i \in I$.

Remark 2.7. *Since coproducts in a category are unique up to isomorphisms, using Theorems 2.3 and 2.6, if* $(F, A)_{\mathcal{A}}$ *is a soft coproduct of* $\{(F_i, A_i)_{\mathcal{A}_i}\}_{i\in I}$ *in* **SoftAct**, *then* A *and* $\mathcal{A}$ *are isomorphic to* $\coprod_{i\in I} A_i$ *and* $\coprod_{i\in I} \mathcal{A}_i$ *in* **Set** *and* **Act-**S, *respectively.*

2.3 ON SOFT EQUALIZERS AND SOFT COEQUALIZERS

This subsection is devoted to study soft equalizers and soft coequalizers in **SoftAct**. We obtain the interrelations of these notions in **SoftAct** with their correspondings in **Act-**S and **Set** under some conditions. But, considering **WSoftAct**, these connections remain as an open problem.

The following result presents a soft equalizer of some soft S-acts.

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Theorem 2.8. *The soft equalizer of soft homomorphisms* $(F_1, A_1)_{\mathcal{A}_1} \overset{(f_1,g_1)}{\underset{(f_2,g_2)}{\rightrightarrows}} (F_2, A_2)_{\mathcal{A}_2}$ *in* **SoftAct**, *under the condition "for every subact* X *of* $\mathcal{A}_1$, *if* $f_1(X) = f_2(X)$, *then* $f_1(x) = f_2(x)$ *for all* $x \in X$", *is a pair* $((F, E)_{\mathcal{E}}, (e, e'))$, *where* $(e, e') : (F, E)_{\mathcal{E}} \to (F_1, A_1)_{\mathcal{A}_1}$ *is a soft homomorphism for which* $(\mathcal{E}, e)$ *and* (E, e') *are the (existing) equalizers of* $\mathcal{A}_1 \overset{f_1}{\underset{f_2}{\rightrightarrows}} \mathcal{A}_2$ *and* $A_1 \overset{g_1}{\underset{g_2}{\rightrightarrows}} A_2$ *in* **Act**-S *and* **Set**, *respectively, and* $F : E \to \mathrm{P}(\mathcal{E})$ *is given by* $F = F_1|_E$.

On soft coequalizers in **SoftAct**, the following result is obtained.

Theorem 2.9. *The soft coequalizer of soft homomorphisms* $(F_1, A_1)_{\mathcal{A}_1} \overset{(f_1,g_1)}{\underset{(f_2,g_2)}{\rightrightarrows}} (F_2, A_2)_{\mathcal{A}_2}$ *in* **SoftAct**, *under the condition "for every subact* X *of* $\mathcal{A}_1$, $f_1(X) = f_2(X)$", *is a pair* $((F, C)_{\mathcal{C}}, (c, c'))$, *where* $(c, c') : (F_2, A_2)_{\mathcal{A}_2} \to (F, C)_{\mathcal{C}}$ *is a soft homomorphism for which* $(\mathcal{C}, c)$ *and* (C, c') *are the coequalizers of* $\mathcal{A}_1 \overset{f_1}{\underset{f_2}{\rightrightarrows}} \mathcal{A}_2$ *and* $A_1 \overset{g_1}{\underset{g_2}{\rightrightarrows}} A_2$ *in* **Act**-S *and* **Set**, *respectively, and* $F : C \to \mathrm{P}(\mathcal{C})$ *is given by* $F([a]_\theta) = c(F_2(a))$, *for every* $a \in A_2$.

2.4 ON SOFT PULLBACKS AND SOFT PUSHOUTS

Finally, we study the notions of soft pullbacks and soft pushouts in **SoftAct** and **WSoftAct**.

In the following, a result concerning w-soft pullbacks in **WSoftAct** under some conditions is obtained.

Theorem 2.10. *Let* $(f_i, g_i) : (F_i, A_i)_{\mathcal{A}_i} \to (F, A)_{\mathcal{A}}, i = 1, 2$, *be weak soft homomorphisms satisfying the condition "for every subacts* X *of* $\mathcal{A}_1$ *and* Y *of* $\mathcal{A}_2$, *if* $f_1(X) = f_2(Y)$, *then* $f_1(x) = f_2(y)$ *for every* $x \in X, y \in Y$". *A w-soft pullback of* $(f_i, g_i), i = 1, 2$, *in* **WSoftAct** *is a pair* $((G, P)_{\mathcal{P}}, ((p_1, p'_1), (p_2, p'_2)))$, *where* $(\mathcal{P}, (p_1, p_2))$ *and* $(P, (p'_1, p'_2))$ *are the (existing) pullbacks of* f_1, f_2 *and* g_1, g_2 *in* **Act**-S *and* **Set**, *respectively, and* $(p_i, p'_i) : (G, P)_{\mathcal{P}} \to (F_i, A_i)_{\mathcal{A}_i}, i = 1, 2$, *are weak soft homomorphisms for which* $G : P \to \mathrm{P}(\mathcal{P})$ *is defined by* $G(a_1, a_2) = F_1(a_1) \times F_2(a_2)$, *for all* $(a_1, a_2) \in P$.

Here we study a particular kind of soft pushouts, so called *soft amalgamated coproduct*, in **SoftAct**.

Problem 2.11. *The soft amalgamated coproduct* $(G, A)_{\mathcal{A}} \coprod^{(F,U)_{\mathcal{U}}} (G, A)_{\mathcal{A}}$ *is the pair* $((H, A \coprod^U A)_{\mathcal{A} \coprod^{\mathcal{U}} \mathcal{A}}, ((q_1, q'_1), (q_2, q'_2)))$, *where* $(A \coprod^U A, q'_1, q'_2)$ *and* $(\mathcal{A} \coprod^{\mathcal{U}} \mathcal{A}, q_1, q_2)$ *are the amalgamated coproducts in* **Set** *and* **Act**-S, *respectively, and* $H(a, i) = q_i(G(a))$ *for every* $a \in A \backslash U$, $i = 1, 2$, $H(a) = F(a)$ *for every* $a \in U$.

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About the Cohen-Macaulay Modules

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Abstract: Let R be a commutative Noetherian ring. A non-zero finitely generated $R-$module M is said to be a Cohen- Macaulay $R-$module precisely when $grade_M I = ht_M I$, for every ideal I of R for which $M \neq IM$. In this paper we are going to study the Cohen-Macaulay modules and a number of results concerning these kinds of modules are given.

Keywords: Noetherian rings, Local rings, Chohen-Macaulay rings, Chohen-Macaulay modules, finitely generated modules.

1 INTRODUCTION

We assume throughout that R is a commutative Noetherian ring and M is a non-zero finitely generated R-module.

Let I be a proper ideal of R. We recall that

$$htI = min\{htP | P \in Spec(R), I \subseteq P\}.$$

In $[2, \oint 48]$, Macaulay showed that, when R is a polynomial ring in finitely many indeterminates with cofficients in a field, and if $htI = n$ and I can be generated by n elements, then $htP = n$ for every $P \in assI$.

In $[1, Theorem 21]$, Cohen established the corresponding result when R is a regular local ring. It turns out that commutative Noetherian ring R is Cohen-Macaulay if and only if, for an $n \in \mathbf{N_0}$, for every proper ideal I of R of height n which can be generated by n elements, we have $htP = n$ for every $P \in assI$.

Let $a_1, ..., a_n \in R$. We know by $[3, 16.1]$, that $a_1, ..., a_n$ form an $M-$sequence of elements of R precisely when

(i) $M \neq (a_1, ..., a_n)M$, and

(ii) For each $i = 1, ..., n$, the element a_i is a non-zerodivisor on the $R-$module $\dfrac{M}{(a_1, ..., a_{i-1})M}$.

Let I be an ideal of R such that $M \neq IM$. We know by $[3, 16.13]$, that every two maximal $M-$sequence in I have the same length. The common length of all maximal $M-$sequences in I denoted by $grade_M I$. If $M = R$, then we show $grade_R I$ by $gradeI$.

For $P \in Supp(M)$, we know by $[3, 17.15]$, that the $M-$height of P , denoted $ht_M P$, is defined by $dim_{R_P} M_P = dim(\dfrac{R_P}{Ann_{R_P}(M_P)})$. Let I be an ideal of R such that $M \neq IM$. We know by $[3, 17.15]$, that the $M-$height of I, denoted $ht_M I$, is defined by

$$ht_M I = min\{ht_M P | P \in Supp(M), I \subseteq P\}.$$

If (R, J) is a local ring, then we show the $grade_M J$ by $depthM$ and $ht_M J$ by $dimM$.

We know by $[3, 17.1]$, that R is a Cohen-Macaulay ring precisely when $gradeI = htI$, for every proper ideal I of R.

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Let R be a commutative Noetherian ring and M be a non-zero finitely generated $R-$module. Let I be an ideal of R such that $M \neq IM$. We know by [3, 9.23] that there exists a prime ideal $P \in Supp(M)$ such that $I \subseteq P$ and by [3, 17.15], we have

$$ht_M I = min\{ht_M P | P \in Supp(M), I \subseteq P\}.$$

Also, we know by [3, 17.16] that M is a Cohen-Macaulay $R-$module precisely when $grade_M I = ht_M I$, for every ideal I of R for which $M \neq IM$.

2 RESULTS

Definition 2.1. *Let R be a commutative Noetherian ring and M be a non-zero finitely generated $R-$module. A proper ideal I of R with $M \neq IM$ is said to be $M-$unmixed when $ht_M I = ht_M P$ for all $P \in Ass(\frac{M}{IM})$.*

Theorem 2.2. *Let R be a commutative Noetherian ring and M be a non-zero finitely generated $R-$module. Then M is Cohen-Macaulay if and only if, for every $k \in N_0$, every proper ideal I of R with $M \neq IM$ and $ht_M I = k$ Which can be generated by k elements is $M-$unmixed.*

Definition 2.3. *Let (R, J) be a commutative Noetherian local ring and M be a non-zero finitely generated $R-$module with $M \neq JM$. We say that M is semi-regular precisely when $depth M = dim M$.*

Definition 2.4. *Let (R, J) be a commutative Noetherian local ring and M be a non-zero finitely generated $R-$module with $M \neq JM$. We say that M is a regular local $R-$module if $dim M = dim_{\frac{R}{J}} \frac{J}{J^2}$.*

Theorem 2.5. *Let (R, J) be a commutative Noetherian local ring and M be a non-zero finitely generated $R-$module. If M be a regular local $R-$module, then it is semi-regular.*

Remark 2.6. *Let (R, J) be a commutative Noetherian local ring and M be a non-zero finitely generated $R-$module. We know by [3, 17.17], that M is Cohen-Macaulay if and only if it is semi-regular.*

Corollary 2.7. *Let (R, J) be a commutative Noetherian local ring and M be a non-zero finitely generated $R-$module. If M be a regular local $R-$module, then it is Cohen-Macaulay.*

Theorem 2.8. *Let R be a commutative Noetherian ring, M be a non-zero finitely generated $R-$module and S be a multiplicatively closed subset of R. If M be a Cohen-Macaulay $R-$module, then M_S is a Cohen-Macaulay R_S-module.*

Remark 2.9. *Let R be a commutative Noetherian ring, M be a non-zero finitely generated $R-$module and S be a multiplicatively closed subset of R. We know by [3, 17.18], That if M_P be a Cohen-Macaulay R_P-module for every $P \in Supp(M)$, then M is a Cohen-Macaulay $R-$module. So the converse of the Theorem 2.8 is true by Corollary 2.10.*

Corollary 2.10. *Let R be a commutative Noetherian ring and M be a non-zero finitely generated $R-$module. If M_S be a Cohen -Macaulay R_S-module, for every multiplicatively closed subset S of R, then M is a Cohen-Macaulay $R-$module.*

Corollary 2.11. *Let R be a commutative Artinian ring. Then every non-zero finitely generated $R-$module is Cohen-Macaulay.*

Lemma 2.12. *Let R be a commutative Noetherian ring and suppose that the non-zero finitely generated $R-$module M is Cohen-Macaulay. Let*

$$P_0 \subset P_1 \subset ... \subset P_n$$

be a saturated chain of prime ideals in $Supp(M)$. Then $ht_M P_n = ht_M P_0 + n$.

Theorem 2.13. *Let R be a commutative Noetherian ring and M be a non-zero finitely generated $R-$module and let $a \in J(R)$ be such that a forms an $M-$sequence. Then M is Cohen-Macaulay if and only if $\frac{M}{(a)M}$ is Cohen-Macaulay.*

Theorem 2.14. *Let R be a commutative Noetherian ring and M be a non-zero finitely generated Cohen-Macaulay $R-$module. Let $a_1, ..., a_n$ be an $M-$sequence of elements of R. Then $\frac{M}{(a_1, ..., a_n)M}$ is again a Cohen-Macaulay module.*

Definition 2.15. *Let (R, J) be a commutative Noetherian local ring and M be a non-zero finitely generated $R-$module with $JM \neq M$. Let $dimM = n$. A system of parameters for M is a set of n elements of J which generates a $J-$primary ideal of R.*

Remark 2.16. *Let R be a commutative Noetherian ring , M be a non-zero finitely generated $R-$module and I be an ideal of R with $IM \neq M$. We know by* [3, 17.15], *that $grade_M I \leq ht_M I$.*

Theorem 2.17. *Let (R, J) be a commutative Noetherian local ring and M be a non-zero finitely generated $R-$module with $JM \neq M$. Set $depthM = d$ and $dimM = n$. Let $a_1, ..., a_d$ be an $M-$sequence in J. Then there exist $a_{d+1}, ..., a_n \in J$ such that $(a_1, ..., a_d, a_{d+1}, ..., a_n)$ is a system of parameters for M.*

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Hamiltonicity of the Unit and Unitary Cayley Graphs

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Abstract: The unit (Unitary Cayley) graph corresponding to an associative ring R is the graph obtained by setting all the elements of R to be the vertices and defining distinct vertices x and y to be adjacent if and only if $x + y \in U(R)$ $(x - y \in U(R))$, where $U(R)$ is the set of all unit elements of R. H. R. Maimani, et al. (C. Lanski and A. Maróti) derive necessary and sufficient conditions for unit (Unitary Cayley) graphs to be Hamiltonian. In this paper, by a constructive method, we give simple proofs for these results.
Keywords: Hamiltonian graphs, Unit graphs, Unitary Cayley Graphs, finite rings.

1 INTRODUCTION

The study of algebraic structure using the properties of graphs has become an exciting research topic in the last twenty years. Let n be a positive integer, and let Z_n be the ring of integers modulo n. R. P. Grimaldi defined a graph $\Gamma(Z_n)$ based on the elements and units of n (cf. [7]). The vertices of $G(Z_n)$ are the elements of Z_n; distinct vertices x and y are defined to be adjacent if and only if $x+y$ is a unit of Z_n.

This corresponding graph is generalized as follows: Let R be a ring and $U(R)$ be the set of unit elements of R. The *unit graph* of R, denoted by $G(R)$, is the graph obtained by setting all the elements of R to be the vertices and defining distinct vertices x and y to be adjacent if and only if $x + y \in U(R)$. For some paper about unit graphs, we refer the reader to [1], [3] and [6].

Let G be a graph, V(G) will denote its vertex and $E(G)$ its edge set. A *path* is an ordered list of distinct vertices $v_0, v_1, ..., v_n$ such that v_{i-1} is adjacent to v_i for $i = 1, 2, ..., n$. A graph G is *connected* if there exists a path between any two distinct vertices of G.

A *cycle* of a graph is a path that the start and end vertices are the same. A *Hamiltonian cycle*— is a spanning cycle in G. A graph is called *Hamiltonian* if it contains a Hamiltonian cycle.

H. R. Maimani, et al. in [6] derive necessary and sufficient conditions for unit graphs to be Hamiltonian. In this paper, we give a simple proof for this result.

The *unitary Cayley* graph of a ring R, denoted by $\Gamma(R)$, is the graph whose vertices set is R, and distinct vertices x and y are defined to be adjacent if and only if $x - y$ is a unit of R. For more information about Cayley graphs, we refer the reader to [3] and [4].

C. Lanski and A. Maróti in [5] give necessary and sufficient conditions for Unitary Cayley

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graphs to be Hamiltonian. In this paper, we also give a simple proof for this result.

2 MAIN RESULTS

Throughout this paper R is finite commutative ring and $J(R)$ is the Jacobson radical of R. In this section we present some lemmas and theorems that will be used in the main theorems. For beginning, we have the following well-known theorems.

Theorem 2.1. *Let R be an arbitrary ring. An element $r \in R$ is a unit of R if and only if $\overline{r} \in R/J(R)$ is a unit of $R/J(R)$.*

Theorem 2.2. *Let R be an arbitrary ring. If $G(R/J(R))$ is Hamiltonian, then $G(R)$ is also Hamiltonian.*

The *cross product* (sometimes also tensor product) of two graphs G and H is a graph denoted $G \times H$, with $V(G \times H) = V(G) \times V(H)$ and $(x, y), (x', y')) \in V(G \times H)$ if and only if x is adjacent to x' in G and y is adjacent to y' in H.

Lemma 2.3. *(See [2]) Let G_1 and G_2 be two Hamiltonian graphs. The graph $G_1 \times G_2$ is Hamiltonian if and only if either $|G_1|$ or $|G_2|$ be odd.*

Lemma 2.4. *Let $R = F_1 \times F_2 \times ... \times F_n$ such that F_i is a field of even order and $|F_i| > 2$ for all $2 \le i \le n$. Then $G(R)$ is a Hamiltonian graph.*

Proof. First suppose that $|F_i| \ge 4$, for all $1 \le i \le n$. We use induction on n. Clearly, for $n = 1$ the assertion is clearly true. Assume that $a_1, a_2, ..., a_r$ is a Hamiltonian cycle in $G(R)$, such that $a_i = (a_{i1}, a_{i2}, ..., a_{in})$, for all $1 \le i \le r$. Let $F_{n+1} = \{f_1, f_2, ..., f_{2k}\}$. For abbreviation, let

$$a_i(f_j) := (a_{i1}, a_{i2}, ..., a_{in}, f_j)$$

Also we define $P_{i,i+1}$ be the following path, $P_{i,i+1} := a_i(f_2), a_{i+1}(f_1), a_i(f_3), a_{i+1}(f_2), ..., a_i(f_{2k}), a_{i+1}(f_{2k-1}), a_i(f_1), a_{i+1}(f_{2k})$

Now we construct the following Hamiltonian cycle for $F_1 \times F_2 \times ... \times F_n \times F_{n+1}$,

$$P_{1,2}, P_{3,4}, ..., P_{r-1,r}$$

.

For the rest part of the proof, it is sufficient to show that $G(F \times Z_2)$, where F is a field and $|F| = 2^m$ for some $m \ge 2$, has Hamiltonian cycle. Let $F = \{f_1, f_2, ..., f_{2^m}\}$. The following, is a Hamiltonian cycle for $G(F \times Z_2)$,
$(f_1, 0), (f_2, 1), (f_3, 0), (f_4, 1), ..., (f_{2^m-1}, 0), (f_2^m, 1)$
$(f_2, 0), (f_1, 1), (f_4, 0), (f_3, 1), ..., (f_{2^m}, 0), (f_{2^m-1}, 1)$. □

Corollary 2.5. *Let $R = F_1 \times F_2 \times ... \times F_n$ such that F_i is a field and $|F_i| > 2$ for all $2 \le i \le n$. Then $G(R)$ is a Hamiltonian graph.*

By an easy observation, we have the following lemma:

Lemma 2.6. *Let R be a ring such that $G(R)$ is Hamiltonian. If $|R|$ is odd, then $G(Z_2 \times R)$ is Hamiltonian.*

By using the before lemmas and theorems, we have the following main results:

Theorem 2.7. *(See [6]) Let R be a ring such that $R \ncong Z_2$ and $R \ncong Z_3$. Then the following conditions are equivalent:*

(1) The graph $G(R)$ is Hamiltonian.

(2) The graph $G(R)$ is connected.

(3) Every element of R is a sum of at most three units.

(4) The ring R is generated by its units.

(5) The ring R cannot have $Z_2 \times Z_2$ as a quotient.

Theorem 2.8. *(See [5]) Let R be a ring. Then the following conditions are equivalent:*

(1) The factor ring $R/J(R)$ has at most one summand isomorphic to Z_2.

(2) Every element of R is a sum of two or three units.

(3) The graph $\Gamma(R)$ is connected.

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بررسی نرمالیته گراف های کیلی روی گروه های متقارن با مجموعه رابطی متشکل ازمجموعه ای از ترانهش ها

نسرین حسینی چگنی فارغ التحصیل کارشناسی ارشد

چکیده: فرض کنید T یک مجموعه از ترانهش‌های گروه متقارن S_n باشد.گراف ترانهشی T گرافی با مجموعه رئوس $\{1, 2, \ldots, n\}$ و مجموعه یال‌های $\{ij \mid (i\ j) \in T\}$ است. در این مقاله نشان داده می شود که اگر $n \geq 3$، آنگاه گروه خودریختی گراف ترانهشی $Tra(T)$ با $Aut(S_n, T)$ یکریخت است که در آن
$Aut(S_n, T) = \{\alpha \in Aut(S_n) \mid T^\alpha = T\}$. هم‌چنین اگر T مجموعه مولد مینیمال S_n باشد، آن‌گاه گراف های کیلی $Cay(S_n, T)$ نرمالند یعنی گروه خودریختی گراف کیلی $Cay(S_n, T)$ نیم ضرب $R(S_n)Aut(S_n, T)$.

کلمات کلیدی: گراف ترانهشی،گراف کیلی،گراف کیلی نرمال.

مقدمه

در این مقاله نرمال بودن گراف‌های کیلی $Cay(S_n, T)$ را بررسی می‌کنیم که در آنها T یک مجموعه مولد مینیمال از ترانهش‌های S_n است. God-sil وRoyle در منبع [4] اثبات کردند که اگر گروه خودریختی گراف ترانهشی $Tra(T)$ بدیهی باشد، آن گاه $Aut(Cay(S_n, T)) = R(S_n)$ که $R(S_n)$ نمایش منظم راست S_n است. ما در این پایان نامه بدون ایجاد هیچ محدودیتی روی گروه خودریختی گراف ترانهشی ثابت می‌کنیم که گراف‌های $Cay(S_n, T)$ نرمالند یعنی $Aut(Cay(S_n, T)) = R(S_n)Aut(S_n, T)$. دربخش اول برخی از مفاهیم اولیه مورد نیاز در مقاله را بیان می کنیم.در بخش دوم به تعدادی لم و گزاره مورد نیاز جهت اثبات قضیه اصلی می پردازیم. در بخش سوم قضیه اصلی مقاله را بیان می کنیم و سپس در قسمت نتایج به بیان چند نتیجه می پردازیم.

بخش یک

در این مقاله مجموعه رئوس،مجموعه یال ها و گروه خودریختی گراف X را به ترتیب با $V(X)$، $E(X)$ و $Aut(X)$ نمایش می دهیم. گروه G را نیم ضرب مستقیم دو گروه KوN نامیده و با NKنمایش می دهیم هرگاه

$1)NG \quad 2)K \leq G \quad 3)N \cap K = 1 \quad 4)G = NK$

اگر G یک گروه باشد و $\emptyset \neq S \subseteq G$ به طوری که $1_G \notin S$ و $S^{-1} = \{s^{-1} \mid\ s \in S\} = S$ ، یعنی S نسبت به وارون بسته باشد، آن‌گاه گراف کیلی $\Gamma = Cay(G, S)$ را به صورت زیر تعریف می‌کنیم $V(\Gamma) = G$ و $E(\Gamma) = \{(g, sg\} \mid\ g \in G\ ,\ \ s \in S\}$ یعنی $\{x, y\} \in E(\Gamma) \Longleftrightarrow xy^{-1} \in S$.
S را مجموعه رابط گراف کیلی Γ گوییم.

بخش دو

گزاره۱:فرض کنید T یک مجموعه از ترانهش‌های S_n باشد و g, h عناصری از T باشند و گراف T شامل هیچ مثلثی نباشد، در این صورت اگر $gh = hg$ آن‌گاه g, h دقیقا دو همسایه در$Cay(S_n, T)$ دارند و اگر $gh \neq hg$ آن‌گاه g, h دقیقا یک همسایه در $Cay(S_n, T)$ دارند. یعنی همانی S_n که آن را e می نامیم.

گزاره۲:یک خودریختی φ از S_n ترانهش‌ها را حفظ می کند؛ اگر و تنها اگر داخلی باشد.

گزاره۳:اگر $n \geq 2$ و $n \neq 6$، آن‌گاه $Aut(S_n) = Inn(S_n)$. اگر $n = 6$، آن گاه $|Aut(S_6) : Inn(S_6)| = 2$. برای هر $\alpha \in Aut(S_6) \setminus Inn(S_6)$، α هر ترانهش را به حاصلضرب سه ترانهش مجزا می‌نگارد.

گزاره۴:فرض کنید $n \geq 3$، و T یک مجموعه از ترانهش های S_n باشد. در این صورت $Aut(Tra(T))$ یکریخت با $Aut(S_n, T)$ است. $Aut(Tra(T)) \cong Aut(S_n, T)$. گزاره برای $n = 2$ درست نیست.اگر T تهی باشد، گزاره برای $n \geq 3$ و $n \neq 6$ برقرار است.

گزاره۵:فرض کنید T یک مجموعه مولد مینیمال از ترانهش‌های S_n باشد. و فرض کنید $X = Cay(S_n, T)$. اگر t , k ترانهش‌های مجزا در T باشند، بنابراین $kt = tk$، اگر و فقط اگر یک دور ۴–تایی منحصر بفرد در X وجود داشته باشد که از e, k, t بگذرد. یعنی $(e, k, tk = kt, t, e)$. علاوه بر این اگر $kt \neq tk$، آنگاه یک دور۶–تایی منحصر بفرد در X وجود دارد که از e, k, t و یک رأس به فاصله‌ی ۳ از e در X بگذرد. یعنی $(e, k, tk, ktk = tkt, kt, t, e)$.

بخش سه

در این بخش به بیان قضیه اصلی و کلیاتی از اثبات آن می پردازیم.فرض کنید T یک مجموعه از ترانهش های S_n باشد. اگر $n \geq 3$، آن‌گاه گروه خودریختی گراف ترانهشیT با $Aut(S_n, T)$ یکریخت است. اگر T مجموعه مولد مینیمال از S_n باشد، آن‌گاه گروه خودریختی گراف کیلی $Cay(S_n, T)$ نیم ضرب $R(S_n)Aut(S_n, T)$ است.

$$Aut(Cay(S_n, T)) \cong R(S_n)Aut(S_n, T)$$

اثبات:ابتدا نشان می دهیم $A = R(S_n)A_e$، سپس ثابت می کنیم $A_e = Aut(S_n, T)$، بنابراین $A = R(S_n)Aut(S_n, T)$. با توجه به [10] داریم:$R(S_n)Aut(S_n, T) = N(R(S_n))$. بنابراین$R(S_n)Aut(S_n, T) \cong Aut(Cay(S_n, T)$ که این یعنی گراف کیلی $Cay(S_n, T)$ نرمال است.

نتایج

نتیجه۱:اگر T یک مجموعه مولد مینیمال از ترانهش های S_n باشد و $Aut(Tra(T)) = 1$، آن‌گاه $Aut(Cay(S_n, T)) \cong S_n$.

نتیجه۲:فرض کنید T یک مجموعه مولد از ترانهش‌های S_n باشد به طوری که برای هر $t_1, t_2 \in T$که $t_1 \neq t_2$، $t_1t_2 = t_2t_1$ اگر و فقط اگر یک دور ۴تایی منحصربه فرد در $Cay(S_n, T)$ وجود داشته باشد که از e و t_1 و t_2 بگذرد و اگر$t_1t_2 \neq t_2t_1$، آن‌گاه یک دور ۶تایی وجود دارد که از e و t_1 و t_2 و یک رأس به فاصله ۳ از e بگذرد. آن‌گاه طبق اثبات قضیه اصلی $R(S_n)Aut(S_n, T) \cong Aut(Cay(S_n, T))$ یعنی $Cay(S_n, T)$ نرمال است. علاوه براین اگر $n \geq 3$ بنابراین $Aut(Tra(T)) \cong Aut(S_n, T)$.

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Prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of semihypergroups

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Abstract: In this paper, we introduce the notion of prime and semiprime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of semihypergroups by using belongs to relation $(\in)$ and quasi-coincidence with relation (q) between intuitionistic fuzzy points and intuitionistic fuzzy sets. Also, we study some properties of these notions.

1 Introduction

After the introduction of fuzzy sets by Zadeh [11], reconsideration of some concepts of classical mathematics began. On the other hand, because of the importance of group theory in mathematics, as well as its many areas of applications, the notion of a fuzzy subgroup was defined and its structure was investigated by Rosenfeld [10]. This subject has been studied further by many mathematicians. A new type of fuzzy subgroup ($(\in, \in \vee q)$-fuzzy subgroup) was introduced by Bhakat and Das [4]. In fact, $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup. Atanassov has introduced and developed the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [3]. Coker and Demirci [5] introduced the notion of intuitionistic fuzzy point. Abdollah et al. [1] introduce the notion of (α, β)-intuitionistic fuzzy ideals of hemirings where α, β are any two of $\{\in, q, \in \vee q, \in \wedge q\}$ with $\alpha \neq \in \wedge q$ and related properties are investigated. In this paper, using the concept of intuitionistic fuzzy point, the notion of prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of semihypergroups is introduced and some related properties are investigated.

2 Preliminaries

A *hypergroupoid* is a non-empty set S together with a map $\cdot : S \times S \longrightarrow \mathcal{P}^*(S)$ where $\mathcal{P}^*(S)$ denotes the set of all the non-empty subsets of S. The image of the pair (x, y) is denoted by $x \cdot y$. If $x \in S$ and A, B are non-empty subsets of S, then $A \cdot B$ is defined by $A \cdot B = \bigcup_{a \in A, b \in B} a \cdot b$. Also $A \cdot x$ is used for $A \cdot \{x\}$ and $x \cdot A$ for $\{x\} \cdot A$. A hypergroupoid $(S, \cdot)$ is called a *semihypergroup* if $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ for all $x, y, z \in S$. A non-empty subset X of a semihypergroup S is called a *subsemihypergroup* if $X \cdot X \subseteq X$. A subsemihypergroup X of S is called a *bi-hyperideal* [2] if

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$X \cdot S \cdot X \subseteq X$.

An *intuitionistic fuzzy set* (IFS for short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \lambda_A(x)\rangle \mid x \in X\}$ where the functions $\mu_A : X \longrightarrow [0,1]$ and $\lambda_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. For the sake of simplicity, we shall use the notation $A = (\mu_A, \lambda_A)$ instead of $A = \{\langle x, \mu_A(x), \lambda_A(x)\rangle \mid x \in X\}$. An IFS $A = (\mu_A, \lambda_A)$ in S is called *intuitionistic fuzzy subsemihypergroup* [6] of S if (1) $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y)$ and (2) $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y)$, for all $x, y \in S$ and $z \in x \cdot y$.

Definition 2.1. [9] An intuitionistic fuzzy subsemihypergroup $A = (\mu_A, \lambda_A)$ of S is called an *intuitionistic fuzzy bi-hyperideal* of S if
(1) $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y)$,
(2) $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y)$,
for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$.

Let X be a non-empty set and $c \in X$ a fixed element in X. A *fuzzy point* c_t (see [8]) for $t \in (0,1]$ is a fuzzy subset of X such that

$$c_t(x) = \begin{cases} t & c = x, \\ 0 & \text{otherwise.} \end{cases}$$

If $t \in (0,1]$ and $s \in [0,1)$ are two fixed real numbers such that $0 \leq t + s \leq 1$, then the IFS $c(t,s) = \langle x, c_t, 1 - c_{1-s}\rangle$ is called an *intuitionistic fuzzy point* (IFP for short) [5] in X, where t (resp. s) is the degree of membership (resp. nonmembership) of $c(t,s)$ and $c \in X$ is the support of $c(t,s)$. An IFP $c(t,s)$ is said to *belong to* an IFS $A = (\mu_A, \lambda_A)$ of X, denoted by $c(t,s) \in A$, if $\mu_A(c) \geq t$ and $\lambda_A(c) \leq s$. We say that $c(t,s)$ is *quasi-coincident* with $A = (\mu_A, \lambda_A)$, denoted by $c(t,s)qA$, if $\mu_A(c) + t > 1$ and $\lambda_A(c) + s < 1$. To say that $c(t,s) \in \vee qA$ means that $c(t,s) \in A$ or $c(t,s)qA$. For any intuitionistic fuzzy set $A = \langle \mu_A, \lambda_A\rangle$ in S and $t \in (0,1], s \in [0,1)$, the sets $U_{(t,s)} = \{x \in S \mid x(t,s) \in A\}$ and $[A]_{(t,s)} = \{x \in S \mid x(t,s) \in \vee qA\}$. are called $\in$-*level set* and $\in \vee q$-*level set* of A, respectively.

Definition 2.2. [7]Let $A = \langle \mu_A, \lambda_A\rangle$ be IFS in S. Then, $A = (\mu_A, \lambda_A)$ is called an $(\in, \in \vee q)$-*intuitionistic fuzzy* bi-hyperideal of S if for all $x, w, y \in S$, ($t_1, t_2 \in (0, 0.5]$ and $s_1, s_2 \in [0.5, 1)$) or ($t_1, t_2 \in (0.5, 1]$ and $s_1, s_2 \in [0, 0.5)$) the following conditions hold:
(1) If $x(t_1, s_1) \in A$, $y(t_2, s_2) \in A \Longrightarrow z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qA$, for all $z \in x \cdot y$,
(2) If $x(t_1, s_1) \in A$, $y(t_2, s_2) \in A \Longrightarrow z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qA$, for all $z \in x \cdot w \cdot y$.

3 Maine results

In what follows let S denote a semihypergroup.

Definition 3.1. An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S is called *semiprime* if for all $x, w \in S$, ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$); $z(t,s) \in A$ implies that $x(t,s) \in \vee qA$, for all $z \in x \cdot w \cdot x$. An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S is called *prime* if for all $x, w, y \in S$, ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$); $z(t,s) \in A$ implies that $x(t,s) \in \vee qA$ or $y(t,s) \in \vee qA$, for all $z \in x \cdot w \cdot y$.

Now, we have a characterization of prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals.

Theorem 3.2. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal $A = \langle \mu_A, \lambda_A\rangle$ of S is prime if and only if, for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$, $\mu_A(x) \vee \mu_A(y) \geq \mu_A(z) \wedge 0.5$, and $\lambda_A(x) \wedge \lambda_A(y) \leq \lambda_A(z) \vee 0.5$.*

Theorem 3.3. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S is semiprime if and only if $\mu_A(x) \geq \mu_A(z) \wedge 0.5$ and $\lambda_A(x) \leq \lambda_A(z) \vee 0.5$, for all $x, w \in S$ and $z \in x \cdot w \cdot x$.*

Example 3.4. (i) Let N be the set of all positive integers. Then, $(N, \odot)$ is a semihypergroup, where $\odot$ is defined by $x \odot y = \{xty \mid t \in 4N\}$ for all

$x, y \in N$. Now define $\mu_A(x) = \begin{cases} 0.8, & \text{if } x \in 2N \\ 0.9, & \text{if } x \notin 2N \end{cases}$ and $\lambda_A(x) = \begin{cases} 0.2, & \text{if } x \in 2N \\ 0.1, & \text{if } x \notin 2N. \end{cases}$

By Theorem 3.2, it is easy to check that $A = \langle \mu_A, \lambda_A \rangle$ is a prime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of $(N, \odot)$.

(ii) Let $S = \{a, b, c, d\}$. Then, $(S, \cdot)$ is a semihypergroup where "$\cdot$" is defined by the following table

$\cdot$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a\}$
d	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a, b\}$

Now, define an intuitionistic fuzzy set $A = \langle \mu_A, \lambda_A \rangle$ on S by $\mu_A(a) = \mu_A(c) = 0.7$, $\mu_A(b) = 0.6$, $\mu_A(d) = 0.8$ and $\lambda_A(a) = 0.2$, $\lambda_A(b) = 0.3$, $\lambda_A(c) = \lambda_A(d) = 0.1$. By Theorem 3.3, it is routine to see that $A = \langle \mu_A, \lambda_A \rangle$ is a semiprime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S.

Theorem 3.5. *The intersection of any family of prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S is a prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S.*

Definition 3.6. A bi-hyperideal P of S is said to be *semiprime* if for all $x, w \in S$; $x \cdot w \cdot x \subseteq P$ implies that $x \in P$. A bi-hyperideal P of S is said to be *prime* if for all $x, w, y \in S$; $x \cdot w \cdot y \subseteq P$ implies that $x \in P$ or $y \in P$.

Finally, we characterize prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals based on $\in$-level sets and $\in \vee q$-level sets.

Theorem 3.7. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal $A = \langle \mu_A, \lambda_A \rangle$ of S is prime (semiprime) if and only if for $0 < t \leq 0.5$ and $0.5 \leq s < 1$, each non-empty $U_{(t,s)}$ is a prime (semiprime) bi-hyperideal of S.*

Theorem 3.8. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal $A = \langle \mu_A, \lambda_A \rangle$ of S is prime (semiprime) if and only if for all ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$) each $[A]_{(t,s)}$ is a prime (semiprime) bi-hyperideal of S.*

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حلقه‌های n-جمعی

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چکیده: حلقه متناهی و غیردوری R را n-جمعی می‌نامیم هرگاه بتوان آن را به‌صورت اجتماع n زیرحلقه آن نوشت ولی به صورت اجتماع کمتر از n زیرحلقه نتوان نوشت. در صورتی‌که R، n-جمعی باشد، nرا با $\sigma(R)$ نشان می‌دهیم. در این مقاله $\sigma(R)$ را برای حلقه‌های از مرتبه p^2 و p^3 به طور کامل محاسبه می‌کنیم.

کلمات کلیدی: حلقه متناهی ، حلقه n جمعی

مقدمه

بنابر مساله‌ای معروف در نظریه گروه‌ها هیچ گروهی را نمی‌توان به صورت اجتماعی از دو زیرگروه سره آن نوشت. در سال ۱۹۲۶ ریاضیدانی ایتالیایی به نام اسکورزا در مرجع [8] ثابت نمود که گروه G قابل نوشتن به صورت اجتماعی از سه زیر گروه متمایز A،BوC است اگر و تنها اگر این سه زیرگروه دارای اندیس 2 در G بوده و $\frac{G}{A\cap B\cap C}$ با $Z_2\times Z_2$ یکریخت است.
با توجه به استدلال اسکورزا می‌توان دید که هیچ حلقه‌ای را نمی‌توان به صورت اجتماعی از دو زیرحلقه سره آن نوشت با وجود این می‌توان حلقه ای چون R ساخت که اجتماع سه زیرحلقه سره آن باشد این حلقه اولین بار توسط روزا در مرجع [1] ارایه گردید. فرض کنیم $R=Z[X]$ حلقه چند جمله‌ایها روی Z باشد. تعریف می‌کنیم:

$$S_1=\{f\in Z[X]:2\mid f(0)\},$$

$$S_2=\{f\in Z[X]:2\mid f(1)\},$$

$$S_3=\{f\in Z[X]:2\mid f(0)+f(1)\}.$$

حال به سادگی می‌توان دید

$$R=S_1\cup S_2\cup S_3$$

و به علاوه $S_1\cap S_2\cap S_3$ ایده‌آلی از R بوده و حلقه $\frac{R}{S_1\cap S_2\cap S_3}$ با حلقه $Z_2\times Z_2$ یکریخت است. بنابراین طبیعی است تا بپرسیم که آیا مشابهی حلقه‌ای برای نتیجه اسکورزا وجود دارد؟
در ادامه به توصیف حلقه‌های متناهی از مرتبه p^2 که p اول است می‌پردازیم. فرض کنیم R چنین حلقه‌ای باشد
گروه جمعیRیک گروه آبلی است لذا حاصل‌ضرب مستقیمی از گروه‌های دوری است. فرض کنید این گروه‌ها دارای مولدهای $g_1,\cdots,g_k$با مرتبه‌های به‌ترتیب$m_1,\cdots,m_k$ باشند. بنابراین عناصر این حلقه توسط k^2 حاصلضرب به شکل

$$g_ig_j=\sum_{t=1}^{k}c_{ij}^{t}g_t \qquad c_{ij}\in Z_{m_t}$$

مشخص می‌شوند. در ادامه تعاریفی از نظریه گروه

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ها را برای نمایش ساختار یک حلقه متناهی بیان می کنیم.برای معرفی حلقه متناهی R مجموعه‌ای از مولدهای $g_1, \cdots, g_k$ از گروه جمعی R را با دوضابطه زیر در نظر می‌گیریم:

الف) $m_i g_i = 0$ که در آن $1 \leq i \leq k$ و m_i نشان‌دهنده مرتبه جمعی g_i است.

ب) $g_i g_j = \sum_{t=1}^{k} c_{ij}^t g_t$ که در آن

$$1 \leq i, j, t \leq k, c_{ij} \in \mathbf{Z}_{m_t}$$

اگر حلقه R نمایشی به صورت بالا داشته باشد می‌نویسیم:

$$R = \langle g_1, \cdots g_k : g_i g_j = \sum_{t=1}^{k} c_{ij}^t g_i \rangle$$

مثال(۱) حلقه $Z_2 + Z_2$ دارای نمایش حلقه‌ای زیر است:

$$Z_2 + Z_2 = \langle a, b : 2a = 2b = 0, a^2 = a, b^2 = b$$

$$ab = ba = 0\rangle$$

و یک میدان متناهی از مرتبه 4 دارای نمایش

$$\langle a, b : 2a = 2b = 0, a^2 = a, ab = b, b^2 = a + b\rangle$$

است. توجه کنیم که اگر گروه جمعی دوری با مولد g باشد ساختار حلقه به طور کامل توسط g^2 تعیین می‌شود. بنابراین:

$$Z_4 = \langle a, 4a = 0, a^2 = a\rangle$$

در ادامه به حلقه‌هایی که دارای گروه جمعی دوری هستند می‌پردازیم. قضیه زیر که در مرجع[5] اثبات شده است در این خصوص حائز اهمیت است.

قضیه ۱: تعداد حلقه‌های دو به دو غیریکریخت با گروه جمعی C_m برابر است با تعداد مقسوم‌علیه های m. به ویژه برای هرمقسوم‌علیه d از m یک حلقه

$$R_d = \langle g : mg = 0, g^2 = dg\rangle$$

وجود دارد که g یک مولد جمعی C_mاست.به ازای d های مختلف این حلقه ها دو به دو غیریکریختند.

نتیجه ۱: اگر p اول باشد در حد یکریختی دقیقا دو حلقه از مرتبه p وجود دارد که عبارتند از Z_p و $C_p(0)$.

نتیجه ۲ : اگر p و q دو عدد اول مجزا باشند در حد یکریختی دقیقا 4 حلقه از مرتبه pq وجود دارد که عبارتند از: Z_{pq} ، $C_{pq}(0)$، $C_p(0) + Z_q$ و$Z_p + C_q(0)$.

در حالت کلی اگر n عددی صحیح و مثبت و خالی از مربع باشد و R حلقه‌ای از مرتبه n آنگاه گروه جمعی R باید دوری باشد.لذا نتیجه زیر بدست می‌آید:

نتیجه ۳: اگر $n = p_1 p_2 \cdots p_k$ یک عدد صحیح مثبت خالی از مربع باشد آن‌گاه درحد یکریختی دقیقا 2^k حلقه از مرتبه n وجود دارد.

قضیه ۲: برای هر عدد اول p در حد یکریختی دقیقا 11 حلقه از مرتبه p^2 وجود دارد.این حلقه‌ها عبارتند از:

$1.A = \langle a | p^2 a = 0, a^2 = a\rangle = Z_{p^2}$

$2.B = \langle a | p^2 a = 0, a^2 = pa\rangle$

$3.C = \langle a | p^2 a = 0, a^2 = 0\rangle = C_{p^2}(0)$

$4.D = \langle a, b | pa = pb = 0, a^2 = a, b^2 = b, ab = 0, ba = 0\rangle = Z_p + Z_p$

$5.E = \langle a, b | pa = pb = 0, a^2 = a, b^2 = b, ab = a, ba = b\rangle$

$6.F = \langle a, b | pa = pb = 0, a^2 = a, b^2 = b, ab = b, ba = b\rangle$

$7.G = \langle a, b | pa = pb = 0, a^2 = 0, b^2 = b, ab = a, ba = a \rangle$

$8.H = \langle a, b | pa = pb = 0, a^2 = 0, b^2 = b, , ab = ba = 0 \rangle = Z_p + C_p(0)$

$9.I = \langle a, b | pa = pb = 0, a^2 = b, ab = 0 \rangle$

$10.J = \{a, b | pa = pb = 0, a^2 = b^2 = 0 \rangle = C_p(0) \times C_p(0)$

$$11.K = \begin{cases} \langle a, b | pa = pb = 0, a^2 = a, b^2 = ja, \\ ab = b, ba = b \rangle \\ j \ is \ not \ square \ in \ Z_p, p \neq 2 \\ \langle a, b | 2a = 2b = 0, a^2 = a, b^2 = a + b \\ , ab = b, ba = b \rangle \\ p = 2 \end{cases}$$

نتایج

در این بخش $\sigma(R)$ را برای حلقه‌های از مرتبه p^2 و p^3 به طور کامل مشخص می‌سازیم. ابتدا تعریف زیر را داریم:

تعریف ۱:حلقه R را دوری می‌نامیم هرگاه گروه جمعی R توسط عنصری چون a تولید شده و هرتوان a را بتوان به صورت مضربی از a نشان داد.

قرارداد:اگر حلقه R دوری باشد قرار می‌دهیم $\sigma(R) = \infty$.

تعریف ۲:فرض کنید R حلقه‌ای غیر دوری است.R را n−جمعی نامیم هرگاه بتوان آن را به صورت اجتماع n زیرحلقه نوشت ولی به صورت اجتماع کمتر از n زیرحلقه نتوان نوشت.

در ادامه $\sigma(R)$ را برای یک حلقه از مرتبه p^2 به طور کامل محاسبه می کنیم.

$10.J = \{a, b | pa = pb = 0, a^2 = b^2 = 0 \rangle = c_p \times c_p$

زیرحلقه های حلقه J عبارتند از:

$s_1 = \langle a \rangle = \{a, 2a, \cdots, (p-1)a\}$

$s_2 = \langle b \rangle = \{b, 2b, \cdots, (p-1)b\}$

$s_3 = \langle a + b \rangle = \{a + b, 2a + b, \cdots, (p-1)a + b\}$

$\vdots$

$s_{p+1} = \langle a + (p-1)b \rangle = \{a + (p-1)b, 2a + (2p-2)b, \cdots, (p-1)a + b\}$

لذا $\sigma(R) = p + 1$.

قضیه ۲:فرض کنید R حلقه‌ای غیر دوری از مرتبه p^2 است.در اینصورت $\sigma(R) \in \{\infty, p + 1\}$.

قضیه ۳:فرض کنید R حلقه‌ای غیر دوری از مرتبه p^3 است.در اینصورت

$$\sigma(R) \in \{\infty, 6, p + 1, 2p, 2p - 1\}$$

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When is $C_{ck}(X)$ a P_c - ideal?

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Abstract: Let $C(X)$ be the ring of all continuous real - valued functions and $C_c(X)$ is the subring of all countinuous real valued functions with countable image .Let $C_{ck}(X) = \{f \in C_c(X) : \text{suppf is compact}\}$, $C_{c\psi}(X) = \{f \in C_c(X) : \text{suppf is pseudocompact}\}$. We proved that if I is an ideal contaned in $C_{c\psi}(X)$, then I is a P_c - ideal if and only if $Coz(I)$ is discrete and I is c-pure. In particular we show that $C_{ck}(X)$ is a P_c - ideal if and only if $C_{ck}(X) = \{f \in C_c(X) : f = 0 \quad \text{except on a finite set}\}$.

Keywords: Ring of continuous functions,functionally countable subring, ideal with compact support, CP-space, P_c-ideal.

1 INTRODUCTION

In this paper , all of spaces are zero - dimentional,T_2 space. $C(X)$ is the ring of all countinuous real-valued functions defined on X, βX is the Stone-Cech compactification of X and $C_c(X)$ is the subring of $C(X)$ consisting of functions whit countable image.For all $f, g \in C_c(X)$, $f \vee g = \frac{f+g+|f-g|}{2} \in C_c(X)$, hence $C_c(X)$ is a sublattice of $C(X)$. For each $f \in C_c(X)$, let $Z(f) = \{x \in X : f(x) = 0\}$, $Coz(f) = X - Z(f)$, $suppf = cl_X Coz(f)$ and $Z_c(X) = \{Z(f) : f_c(X)$.It is manifest that $Z_c(X)$ is closed under the countable intersection and finite union and if $f, g \in C_c(X)$ then $Z(f) \cup Z(g) = Z(fg)$, $Z(f) \cap Z(g) = Z(f^2 + g^2)$. It is obvious every set in $Z_c(X)$ is closed in X. If all elements of $Z_c(X)$ are open too, then X is called a countably P-space (briefly, CP-space). Furthermore X is a CP-space iff $C_c(X)$ is a regular ring, i.e., for every $f \in C_c(X)$, there exists $g \in C_c(X)$ such that $f^2 g = f$, see[2].

In $C(X)$, we assume that X is a completely regular space, similarly, if X is any space(not necessarily completely regular), then there is a zero - dimentional space Y(i.e., a T_1-space with a base consisting of clopen sets) which is a continuous image of X and $C_c(X) \simeq C_c(Y)$, for more information see [2,3].

2 Some properties of $C_{ck}(X)$

Difinition2.1: For a topological space X we put

$$C_{ck}(X) = \{f \in C_c(X) : \text{suppf is compact}\}$$

It is manifest that $C_{ck}(X)$ is both an ideal in $C_c(X)$ and $f \in C^*_c(X)$, and if X is compact, then $C_{ck}(X) = C_c(X)$. The following example showes that the equallity in $C_{ck}(X) \subseteq C_k(X) = \{f \in C(X) : \text{suppf is compact}\}$ is not necessarily holds.
Example2.2: Let $X = R$ with natural topology.

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Define $f : R \to R$, such that

$$f(x) = \begin{cases} 0 & : |x| > 1 \\ 1+x & : -1 \leq x < 0 \\ 1-x & : 0 \leq x \leq 1 \end{cases}$$

we have $f \in C(R)$, $Z(f) = (-\infty, 1) \cup (1, \infty)$, $cl(R - Z(f)) = [-1, 1]$ is compact in R. Thus $f \in C_k(R)$, but $f \notin C_{ck}(R)$.
We recall that the ideal I of $C_c(X)$ is called Z_c-ideal, if $Z(f) \in Z[I]$ then $f \in I$. I is called free, if $\bigcap_{f\in I} Z(f) = \phi$
Lemma 2.2: $C_{ck}(X)$ is a Z_c - ideal in $C_c(X)$.
Lemma 2.3: Let I be an ideal in $C_c(X)$. I is free iff for every compact set $A \subseteq X$, there exists $f \in I$ such that $f(x) \neq 0$, for all $x \in A$.
Lemma 2.4: $C_{ck}(X)$ is contained in every free ideal in $C_c(X)$ and in every free ideal in $C_c^*(X)$.
For $p \in \beta X$, we define

$$O_c^p = \{f \in C_c(X) : p \in int_{\beta X} cl_{\beta X} Z(f)\}$$

It is clear that O_c^p is an ideal in $C_c(X)$.
Theorem2.4: $C_{ck}(X)$ is the same as the family of all $f \in C_c^*(X)$ such that $Z_{\beta X}(f^\beta)$ is a neighborhood of $\beta X - X$. Thus

$$C_{ck}(X) = \bigcap_{p\in\beta X - X} O_c^p = O^{\beta X - X}$$

hence $C_{ck}(X)$ is the intersection of all the free ideals in $C_c(X)$, and all the free ideals in $C_c^*(X)$.

3 c-pure and P_c-ideals

Difinitions3.1: Let I be an ideal in $C_c(X)$. I is called c-pure if for each $f \in I$ there exists $g \in I$ such that $f = fg$, and in this case $g = 1$ on $suppf$.
A nonzero ideal I of $C_c(X)$ is called a P_c-ideal if every proper prime ideal of I is maximal in I.
The ideal I is called regular if for each $f \in I$ there exists $g \in I$ such that $f = f^2 g$.
Theorem3.2: Let I be a nonzero ideal of $C_c(X)$. Then the following statemants are equivalent:
(1) I is a P_c-ideal.
(2) $Z(f)$ is open for each $f \in I$.
(3) Every ideal of I is c-pure.
(4) Every prime ideal of $C_c(X)$ which dose not contain I, is maximal in $C_c(X)$.

If X is a CP-space, then every ideal of $C_c(X)$ is a P_c-ideal. In the following there is an example of X is not a CP-space.
Example3.3: Let $X = N^*$ (one-compactification of N). Difine $I = \{f \in C_c(X) : f = 0$ except on a finite set$\}$.It is clear that I is an ideal in $C_{ck}(X)$and I is a P_c-ideal, because for every $f \in C_c(X)$, $Coz(f)$ is finite and then is clopen. Hence $Z(f)$ is clopen, but X is not CP-space.

For ideal I of $C_c(X)$, we put $CozI = \bigcup_{f\in I} Cozf$, and for $x \in X$,

$$M_x^c = \{f \in C_c(X) : x \in Z(f)\}$$
$$O_x^c = \{f \in C_c(X) : x \in intZ(f)\}$$

M_x^c and O_x^c are ideals in $C_c(X)$, $O_x^c \subseteq M_x^c$.(see [ ])
Theorem3.4: Let I be a nonzero ideal of $C_c(X)$. Then the following statemants are equivalent:
(1) I is a P_c-ideal.
(2) $M_x^c = O_x^c$ for each $x \in CozI$ and $I \subseteq O_x^c$ for $x \notin CozI$.
(3) I is a regular ring.
(4) $CozI$ is a CP-space and I is c-pure.

4 When is $C_{ck}(X)$ a P_c - ideal?

We define $C_{c\psi}(X) = \{f \in C_c(X)$: suppf is pseudocompact$\}$. It is manifest $C_{c\psi}(X)$ is an ideal of $C_c(X)$ and $C_{ck}(X) \subseteq C_{c\psi}(X)$.
Lemma 4.1: If I is a c-pure ideal, then $suppf \subseteq CozI$ for $f \in I$.
Theorem4.2: Let I be an ideal contained in $C_{c\psi}(X)$. Then I is a P_c-ideal iff $CozI$ is discrete and I is c-pure.
corollary4.2: The following statements are equivalent:
(1) $C_{ck}(X)$ is a P_c-ideal.
(2) $CozC_{ck}(X)$ is discrete and $C_{ck}(X)$ is c-pure.

(3) $C_{ck}(X) = \{f \in C_c(X) : f = 0$ except on a finite set$\}$.

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درباره ۲-همبندی گراف توان گروه‌های ساده

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چکیده: گراف توان یک گروه متناهی، گرافی است که مجموعه رئوس آن عناصر گروه می‌باشد و دو رأس g و h از این گراف مجاورند اگر و تنها اگر یکی برابر توانی از دیگری باشد. این مقاله به مطالعه ۲-همبندی گراف‌های توان گروه‌های ساده پراکنده، گروه‌های ساده ری نوع $^2F_4(q)$ و گروه‌های تصویری $PSL(3,q)$ می‌پردازد.

کلمات کلیدی: گراف توان، گروه ساده پراکنده، گروه ساده ری نوع $^2F_4(q)$، گروه تصویری $PSL(3,q)$، ۲-همبندی.

مقدمه و مفاهیم اولیه

فرض کنید G یک گروه متناهی است. گراف توان $P(G)$ گرافی است با مجموعه رئوس G که در آن دو عنصر متمایز x و y مجاورند اگر و تنها اگر یکی از آن‌ها توانی از دیگری باشد.

گراف توان سره که با حذف رأس همانی از $P(G)$ به‌دست می‌آید با $P^*(G)$ نشان داده می‌شود. در واقع $P^*(G) = P(G) - \{e\}$. درجه x در گراف توان را با $deg(x)$ و مجموعه مرتبه‌های عناصر G را با $\pi_e(G)$ نشان می‌دهند. رأس برشی در یک گراف، رأسی است که حذف آن موجب افزایش تعداد مولفه‌های همبندی گراف می‌گردد. گراف Γ، ۲-همبند گفته می‌شود هرگاه رأس برشی نداشته باشد.

لم ۱. *([۱]) گراف توان $P(G)$ کامل است اگر و تنها اگر برای هر دو زیرگروه دوری G_1 و G_2 از G داشته باشیم $G_2 \subseteq G_1$ یا $G_1 \subseteq G_2$.*

این مسئله که چه هنگام گراف توان یک گروه دوری کامل است از جمله مسائلی است که بلافاصله بعد از تعریف این گراف به ذهن متبادر می‌شود. در قضیه زیر چاکرابارتی ([۱])، و دیگران به این سئوال پاسخ داده‌اند.

قضیه ۲. *گراف توان $P(G)$ کامل است اگر و تنها اگر G یک $p-$گروه دوری باشد.*

به سادگی می‌توان دید که اگر G یک گروه متناهی و $x \in G$ آن‌گاه

$$deg(x) = |\{g \in G/ <g> \leq <x> \text{ یا } <g> << x>\}| - 1$$

در ادامه نیاز داریم تا روابطی برای محاسبه درجه رئوس در گراف توان داشته باشیم. یکی از این روابط که بسیار مورد استفاده ما خواهد بود نتیجه‌ای از پورقلی و دیگران است که در ([۵]) به اثبات رسیده است.

لم ۳. *فرض کنید Z_n گروه دوری از مرتبه n باشد. در این‌صورت داریم:*

$$deg(x) = o(x) + \sum_{ko(x)|n, k\neq 1} \varphi(ko(x)) - 1$$

نتیجه زیر از چاکرابارتی و دیگران ([۱])، در خصوص تعداد یال‌های گراف توان از جمله نتایج زیبایی است که مسائل زیادی حول و حوش آن بوجود آمده است.

نتیجه ۴. *فرض کنید G گروه متناهی از مرتبه n باشد. در این‌صورت*

$$2e = \sum_{a\in G}(2o(a) - \varphi(o(a)) - 1)$$

که در آن e تعداد یال‌های گراف و $o(a)$ مرتبه رأس a است.

توجه کنید که همبندی $P^*(G)$ با ۲-همبندی $P(G)$ معادل است. ما در این مقاله به جای ۲-همبندی $P(G)$ به همبندی $P^*(G)$ می‌پردازیم.

چند مثال

این بخش به محاسبه گراف توان چند دسته معروف از گروه‌ها اختصاص دارد. به طور دقیق گراف‌های توان گروه‌های دو وجهی، دو دوری و نیم دو وجهی محاسبه خواهند شد. گروه دو وجهی به صورت زیر معرفی می‌شود: $D_{2n} =< a, b \mid a^n = b^2 = 1, bab = a^{-1} >$ به سادگی می‌توان دیدکه گراف توان گروه فوق اجتماع n نسخه از K_4 و گراف توان Z_n است.

گروه نیم دو وجهی SD_{8n} با نمایش زیر معرفی می‌شود:

$$SD_{8n} =< a, b \mid a^{4n} = b^2 = 1, bab = a^{2n-1} > .$$

در این صورت گراف توان گروه فوق اجتماع $2n$ نسخه از K_2، n نسخه از K_4 و گراف توان Z_{4n} است به طوری‌که K_4 ها و $P(Z_{4n})$ در یک یال چنان مشترکند که یک رأس آن یال، همانی و رأس دیگر آن a^{2n} می‌باشد.

حال گروه دو دوری T_{4n} در نظر می‌گیریم. این گروه با نمایش زیر ساخته می‌شود:

$$T_{4n} =< a, b \mid a^{2n} = 1, a^n = b^2, aba = b > .$$

در این صورت گراف توان گروه فوق اجتماع n کپی از K_4 و گراف توان Z_{2n} است که همگی در یک یال مشترکند به طوری‌که یک رأس یال، همانی و رأس دیگر آن عنصر منحصربفرد از مرتبه ۲ در T_{4n} می‌باشد.

نتایج اصلی

پورقلی و دیگران در مرجع [۵] مسئله وجود گروه‌های ساده‌ای که گراف توان آن‌ها ۲-همبند است را مطرح ساختند. در این بخش ثابت می‌شود که گراف توان تمامی گروه‌های ساده پراکنده ۲-همبند نیستند. به علاوه گروه‌های ساده ری نوع ${}^2F_4(q)$ مورد بررسی قرار خواهد گرفت.

بنابر رده بندی گروه‌های ساده، ۲۶ گروه وجود دارند که در هیچ دسته نامتناهی از گروه‌های ساده قرار نمی‌گیرند. این گروه‌ها را گروه‌های ساده پراکنده می‌نامند. گروه‌های ساده پراکنده عبارتند از:

$$\left\{\begin{array}{c} M_{11}, M_{12}, M_{22}, M_{23}, M_{24}, J_1, J_2, J_3, \\ J_4, Co_1, Co_2, Co_3, Fi_{22}, Fi_{23}, Fi_{24}, HS, \\ Msl, He, Ru, SUZ, O'N, HN, Ly, Th, B, M \end{array}\right\}$$

می‌دانیم در گراف توان یک گروه در صورتی میان دو رأس یال وجود دارد که یکی به صورت توانی از دیگری قابل نوشتن باشد. حال با توجه به این‌که در صورت متباین بودن مرتبه دو رأس یالی میان آن دو وجود ندارد، گراف توان گروه‌های پراکنده را در نظر می‌گیریم.

جدول ۱. گروه‌های ساده پراکنده همراه با تعدادی عامل اول متباین با سایر مرتبه عناصر گروه

M_{11}	M_{12}	M_{22}
11	11	11
M_{23}	M_{24}	J_1
23	23	19
J_2	J_3	J_4
7	19	$\{23, 29, 31, 37, 43\}$
Co_1	Co_2	Co_3
23	$\{11, 23\}$	23
He	HN	Mcl
17	19	11
$O'N$	Ly	Ru
31	67	29
Hs	Th	Suz
$\{7, 11\}$	$\{13, 19, 31\}$	$\{11, 13\}$
B	M	Fi_{22}
$\{31, 47\}$	$\{41, 71\}$	$\{13, 17\}$
Fi_{23}	Fi_{24}	–
$\{17, 23\}$	29	–

با یک بررسی ساده روی محاسبات داده شده در جدول ۱، می‌توان دید که در میان مجموعه مرتبه‌های کلاس های تزویج هر یک از گروه‌های پراکنده فوق هیچ عددی یافت نمی‌شود که اعداد مشخص شده زیر هر گروه را عاد کند و یا توسط آن عاد شود. بنابراین قضیه زیر را داریم:

قضیه ۵. *گراف توان گروه‌های پراکنده، ۲-همبند نیستند.*

با محاسباتی خسته کننده و استفاده از نتایج دنگ و شی ([۳])، قضیه زیر به اثبات می‌رسد.

قضیه ۶. *گراف توان گروه‌های ری نوع $^2F_4(q)$، ۲-همبند نیستند.*

دوست آبادی و فرخی ([۴])، ثابت نمودند گروه‌های $PSL(2,p^m)$، دارای $\frac{(p^{2m+1}-1)}{p-1}$ مولفه همبندی هستند. حال طبیعی است که به گروه‌های $PSL(3,q)$ بپردازیم. با استفاده از نتایج درفشه و دیگران ([۲])، می‌توان ۲-همبندی گروه‌های $PSL(3,q)$ را نیز بررسی نمود. به طور دقیق:

قضیه ۷. *گراف توان گروه‌های $PSL(3,q)$، ۲-همبند نیستند.*

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يك بررسي از هم متناهي بودن مدول هاي كوهمولوژي موضعي تعميم يافته

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چكيده: فرض كنيد I يك ايده آل از حلقه موضعي، نوتري و جابجايي R باشد و M و N دو مدول با توليد متناهي باشند. فرض كنيد t يك عدد صحيح مثبت باشد. ما مي دانيم كه اگر به ازاي هر $t > i$ ، $H_I^i(M,N)$ با توليد متناهي باشد، آنگاه $Hom(R/I, H_I^t(M,N))$ با توليد متناهي است، به عنوان يك نتيجه مشابه، ما اساساً ثابت مي كنيم كه اگر به ازاي هر $i < t$ ، $H_I^i(M,N)$ آرتيني باشد، آنگاه به ازاي هر $i < t$ ، $H_I^i(M,N)$ I- هم متناهي است و $Hom(R/I, H_I^t(M,N))$ با توليد متناهي است.

كلمات كليدی:

آرتينی، كوهمولوژی موضعی، هم متناهی.

مقدمه

فرض كنيد R يك حلقه نوتري جابجايي باشد و I يك ايده آل سره از R باشد. در سال ۱۹۶۹، گروتنديك [۱] ، فرضيه زير را مطرح كرد.

۱-۱-فرضيه. فرض كنيد N يك R- مدول با توليد متناهي باشد و I يك ايده آل از R باشد، آنگاه به ازاي هر $i \geq 0$ ، $Hom\left(R/I, H_I^i(N)\right)$ با توليد متناهي است.

هارتشورن [۲] در [8] يك مثال نقض بر اين فرضيه بيان كرد. او يك R- مدول L را I- هم متناهي در نظر گرفت اگر $Supp_R(L) \subseteq V(I)$ و به ازاي هر $i \geq 0$ ، $Ext_R^i(R/I, L)$ يك R- مدول با توليد متناهي باشد. (كه در آن $V(I)$ مجموعه همه ايده آل هاي اول R، شامل I است.) سپس مسئله زير را مطرح كرد.

۱-۲-مسئله. فرض كنيد N يك R- مدول با توليد متناهي باشد و I يك ايده آل از R باشد. آيا به ازاي هر i ، $H_I^i(N)$ ، I- هم متناهي است؟ در حالت كلي، حتي اگر R يك حلقه موضعي منظم باشد، جواب اين سوال منفي است. (براي يك مثال نقض [5] را ببينيد.) از طرف ديگر هرزوگ [۳] در [9] به ازاي هر R - مدول M و N ، مدول كوهمولوژي موضعي تعميم يافته را به صورت زير تعريف كرد.
$H_I^i(M,N) = \lim\limits_{\overrightarrow{n}} Ext_R^i(M/I^nM, N)$ واضح است كه اين تعميم يك مدول كوهمولوژي موضعي معمولي است، مطالعه درباره مدول هاي كوهمولوژي موضعي

[۱] Grothendieck
[۲] Hartshorne
[۳] Herzog

بوسيله بسياري از نويسندگان دنبال شد. (براي نمونه ۲-۱۶ در [13] را ببينيد) كه در آن ياسمي اين سوال را مطرح كرد كه مسئله ۲-۱ براي كوهمولوژي موضعي تعميم يافته برقرار است؟ همچنين هم متناهي بودن مدول هاي كوهمولوژي موضعي تعميم يافته توسط ديواني آذر و سازيده در [7] و خشيار منش و ياسي در [10] مطالعه شده است. از طرف ديگر فرضيه ۱-۱ ما را به طرح مسئله ديگري سوق مي دهد.

۱-۳-مسئله. فرض كنيد M و Nهر R- مدول با توليد متناهي باشند. چه موقع $Hom(R/I, H_I^i(M,N))$، با توليد متناهي است؟

اسدالهي، خشيارمنش و سالارين در [1] ثابت كردند كه اگر به ازاي هر $t > i$ ، $H_I^i(M,N)$ با توليد متناهي باشد، آنگاه $Hom(R/I, H_I^t(M,N))$ با توليد متناهي خواهد بود. به عنوان يك نتيجه مشابه ما نشان مي دهيم كه اگر به ازاي هر $t > i$ ، $H_I^i(M,N)$ آرتيني باشد آنگاه $H_I^t(M,N)$ I - هم متناهي است و $Hom(R/I, H_I^t(M,N))$ با توليد متناهي است. خواننده را به [2] جهت يافتن اصطلاحات يا تعاريف ديگر ارجاع مي دهيم.

شرايط معادل براي هم متناهي بودن مدول هاي آرتيني

۲-۱-لم. فرض كنيد M يك R- مدول با توليد متناهي باشد. اگر $H_I^i(M)$ به ازاي هر $t > i$ ، آرتيني باشد، آنگاه $H_I^t(M)$ ، I- هم متناهي است و $Hom(R/I, H_I^t(M))$ با توليد متناهي است.

برهان: اثبات اين لم به راحتي از قضيه ۲-۱ از [5]و گزاره ۴-۳ از [8] بدست مي آيد.

۲-۲-لم. فرض كنيد M يك R- مدول با توليد متناهي باشد. اگر Lيك R- مدول آرتيني و I- هم متناهي باشد، آنگاه به ازاي هر i، $Ext_R^i(M,L)$ ، I- هم متناهي است.

برهان: چون L آرتيني است، به ازاي هر i، $Ext_R^i(M,L)$ نيز آرتيني است. بنا به گزاره ۴-۳ از [8] كافيست ثابت كنيم كه با توليد متناهي است.

۲-۳-لم. فرض كنيد M يك R- مدول با توليد متناهي باشد و s يك عدد صحيح نامنفي باشد و فرض كنيد L يكR - مدول باشد به طوريكه به ازاي هر $s > i$ ، $H_I^i(M)$ آرتيني و I- هم متناهي باشد. اگر به ازاي هرs > i$ ، $H_I^i(M,L)$ آرتيني باشد، آنگاه$H_I^i(M,L)$ ، I- هم متناهي است.

برهان: با استقرا روي s ثابت مي شود.

۲-۴-لم. فرض كنيد دنباله $0 \to L_1 \to L_2 \to L_3 \to 0$ يك دنباله دقيق از R- مدول هاي با توليد متناهي باشند. آنگاه دنباله زير دقيق است:

$$\ldots \to H_I^i(L_3,N) \to H_I^i(L_2,N)$$
$$\to H_I^i(L_1,N) \to H_I^{i+1}(L_3,N) \to \ldots$$

برهان: فرض كنيد $0 \to N \to E^\bullet$ يک تجزيه انژكتيو مينيمال ازN باشد،(اگرEانژكتيو باشد، $\Gamma_I(E)$ انژكتيو است). آنگاه $Hom(-, \Gamma_I(E))$ يك فانكتور [۴] دقيق است. بنابراين

$$0 \to Hom(L_3, \Gamma_I(E^\bullet)) \to Hom(L_2, \Gamma_I(E^\bullet))$$
$$\to Hom(L_1, \Gamma_I(E^\bullet)) \to 0$$

يك دنباله دقيق ازR- مختلط هاست و از آن دنباله دقيق زير را داريم:

$$\ldots \to H^i(Hom(L_3, \Gamma_I(E^\bullet)) \to$$
$$H^i(Hom(L_2, \Gamma_I(E^\bullet)) \to$$
$$H^i(Hom(L_1, \Gamma_I(E^\bullet)) \to$$
$$H^{i+1}(Hom(L_3, \Gamma_I(E^\bullet)) \to \ldots$$

[۴] Functor

اگر فرض کنیم M يك R- مدول با توليد متناهي باشد، آنگاه

$\Gamma_I(Hom(M,E^\bullet)) \cong Hom(M,\Gamma_I(E^\bullet))$ همچنين

$$H_I^i(M,N) = \varinjlim_n Ext_R^i(M/I^nM,N)$$

$$\cong H^i\left(\Gamma_I(Hom(M,E^\bullet))\right)$$

$$\cong H^i\left(Hom(M,\Gamma_I(E^\bullet))\right)$$

بنابراين حكم ثابت است.

حال در اينجا در موقعيتي قرار داريم كه مي توان نتيجه اصلي مقاله را اثبات نمود.

۵-۲-قضيه. فرض كنيد M و N دو R- مدول با توليد متناهي باشند اگر به ازاي عدد صحيح نامنفي t، و به ازاي هر $t < i$ ، $H_I^i(M,N)$ آرتيني باشد، آنگاه $H_I^i(M,N)$ ، I- هم متناهي است و $Hom(R/I,H_I^t(M,N))$ با توليد متناهي است.

برهان: بنا به قضيه ۲-۲ از [3] چون به ازاي هر $t < i$ ، $H_I^i(M,N)$ آرتيني است، پس

$H_{I+AnnM}^i(N)$ آرتيني است. بنا به لم ۱-۲، به ازاي هر $t < i$ ، $H_{I+AnnM}^i(N)$ ، $(I+AnnM)$ - هم متناهي است. بنابراين طبق لم ۳-۲، $H_{I+AnnM}^i(N)$ ، $(I+AnnM)$ - هم متناهي است. بويژه به ازاي هر $t < i$ ، $Hom\left(R/(I+AnnM), H_{I+AnnM}^i(M,N)\right)$ با توليد متناهي است. (توجه كنيد كه

$$Hom(R/I,H_I^i(M,N)) \cong$$
$$Hom(R/(I+AnnM),H_{I+AnnM}^i(M,N))$$

بنابراين به ازاي هر $t < i$،

$Hom(R/I,H_I^i(M,N))$ با توليد متناهي است.)

پس بنا به گزاره ۳-۴ از [۸] و بنا به فرض اينكه به ازاي هر $t < i$، $H_I^i(M,N)$

آرتيني است نتيجه مي شود كه $H_I^i(M,N)$

آرتيني است. در ادامه كافيست نشان دهيم كه $Hom(R/I,H_I^t(M,N))$ با توليد متناهي است. چون به ازاي هر i، $H_{I+AnnM}^i(M,N) \cong H_I^i(M,N)$ و

$$Hom(R/I,H_I^i(M,N)) \cong$$
$$Hom(R/(I+AnnM),H_{I+AnnM}^i(M,N))$$

مي توان فرض كرد كه $AnnM \subseteq I$ فرض كنيد $0 \to K \to R^n \to M \to 0$ يك دنباله دقيق از R- مدول هاي با توليد متناهي باشد. طبق لم ۴-۲ دنباله دقيق زير را داريم

$$\ldots \to H_I^{t-1}(K,N) \xrightarrow{\alpha}$$
$$H_I^t(m,N) \to H_I^t(R^n,N) \to \ldots$$

واضح است كه به ازاي هر i، $H_I^i(R^n,N) \cong \overset{n}{\underset{i=1}{\oplus}} H_I^i(N)$

. بنابراين طبق قضيه ۲-۲ از [۳] به ازاي هر $t < i$، $H_I^i(K,N)$ و $H_I^i(R^n,N)$ آرتيني هستند.

با استفاده از بخش قبل، مي دانيم كه به ازاي هر $t < i$، $H_I^i(K,N)$، I- هم متناهي است. فرض كنيد L تصوير α در دنباله دقيق بالا باشد، طبق نتيجه ۴-۴ از [۸] L، I- هم متناهي است. حال از دنباله دقيق

$$0 \to L \xrightarrow{\alpha} H_I^t(M,N)$$
$$\to H_I^t(R^n,N) \to \ldots$$

دنباله دقيق زير را داريم

$$0 \to Hom(R/I,L) \to$$
$$Hom(R/I,H_I^t(M,N)) \to$$
$$Hom(R/I,H_I^t(R^n,N))$$

و طبق لم ۱-۲، بخش راست دنباله دقيق بالا با توليد متناهي است، بنابراين نتيجه از دنباله دقيق بالا حاصل مي شود.

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Injective Valuation Modules

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Abstract: In this paper we generalize the notion of valuation to an injective R-module and obtain results which characterize it.

Keywords: multiplication module, valuation module, divisible module, injective module.

1 INTRODUCTION

Throughout this paper, R denotes an integral domain, with quotient field K, $T = R - \{0\}$ and M is a unitary R-module. An R-module M is called a multiplication R-module, if for each submodule N of M, there exists an ideal I of R such that $N = IM$. (For more information about multiplication modules, see [1, 3].) An integral domain R is called a valuation ring, if for each $x \in K - \{0\}$, $x \in R$ or $x^{-1} \in R$ (see [4]). An R-module E is said to be injective if given R-modules $A \subseteq B$ such that $h : A \to B$ be a monomorphism and $f : A \to E$ be a homomorphism, there exists a homomorphism $g : B \to E$ such that $gh = f$. Let R be an integral domain. An R-module D is divisible if for every $d \in D$ and every $0 \neq r \in R$ there exists $c \in D$ such that $rc = d$. In [2] valuation modules is introdused and we prove some interesting results for injective valuation modules.

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2 Valuation Modules

Let R be an integral domain with quotient field K and M a torsionfree R-module. For $y = \frac{r}{s} \in K$ and $x \in M$, we say that $yx \in M$ if there exists $m \in M$ such that $rx = sm$.

Lemma 2.1. *[2] Let R be an integral domain with quotient field K and M a torsionfree R-module. Then the following conditions are equivalent:*

1) *For all $y \in K$ and all $x \in M$, $yx \in M$ or $y^{-1}M \subseteq M$;*

2) *For all $y \in K$, $yM \subseteq M$ or $y^{-1}M \subseteq M$.*

Definition 2.2. *Let R be an integral domain with quotient field K. A torsionfree R-module M is called valuation module (VM) if one of the condition of lemma 2.1 holds.*

Example 2.3. *(i) Any vector space is a valuation module.*

(ii) Let R be a domain. R is a valuation ring if and only if R is a valuation R-module.

Proposition 2.4. *Let K be the quotient field of a domain R and M a torsionfree R-module. Let S be the set, ordered by inclusion, of all non-empty subsets of M. Then the following conditions are equivalent:*

1) M is a valuation module;

2) $S' = \{(N : M)|N \in S\}$ is totally ordered;

(3) For $U = \{rM|r \in R\}$ the subset of S, U' is totally ordered.

Corollary 2.5. *Let R be a domain and M a faithful multiplication R-module. Then M is a valuation module if and only if for any two submodules N, L of M, $N \subseteq L$ or $L \subseteq N$.*

Lemma 2.6. *Let R be a valuation ring and M a torsionfree R-module. Then M is a valuation R-module.*

Corollary 2.7. *Let R be a valuation ring, M a torsionfree R-module and N be a submodule of M. Then N is a valuation R-module.*

Lemma 2.8. *If M is a multiplication valuation R-module, then M is finitely generated and R is a valuation ring.*

Corollary 2.9. *Let M be a multiplication valuation module. Then for each $p \in spec(R)$, M_p is a multiplication valuation module.*

Lemma 2.10. *Let R be a valuation domain. Then every finitely generated torsion-free R-module is free.*

Corollary 2.11. *Let M be a multiplication valuation R-module. Then any finitely generated submodule of M is cyclic.*

Theorem 2.12. *Let M be a finitely generated module over an integrally closed ring R. If M is a valuation module, then M is a free R-module and R is a valuation ring.*

3 Injective Valuation Modules

Definition 3.1. *An R-module E is said to be injective if for any module monomorphism $h : A \longrightarrow B$ and any homomorphism $f : A \to E$, there exists a homomorphism $g : B \to E$ such that $gh = f$.*

Definition 3.2. *Let R be an integral domain. A R-module D is divisible if for every $d \in D$ and every $0 \neq r \in R$ there exists $c \in D$ such that $rc = d$.*

Example 3.3. *$\mathbb{Q}$ is a $\mathbb{Z}$-module divisible.*

Theorem 3.4. *Every R-module injective is divisible.*

Theorem 3.5. *Let R be a principal ideal domain. Then an R-module M is injective if and only if it is divisible.*

Example 3.6. *Any vector space is an injective valuation module.*

Proposition 3.7. *Let M be an divisible valuation R-module. Then For any submodule N of M, such that $\frac{M}{N}$ is a torsionfree R-module, $\frac{M}{N}$ is a divisible valuation module.*

Corollary 3.8. *Let M be an injective valuation R-module. Then For any submodule N of M, such that $\frac{M}{N}$ is a torsionfree R-module, $\frac{M}{N}$ is a injective valuation module.*

Proposition 3.9. *Let R be an artinian semisimple valuation ring and M an injective torsion-free R-module. Then any submodule N of M is an injective valuation module.*

Proposition 3.10. *Let M be an injective valuation module. Then if M' is a torsionfree R-module and $\varphi : M \to M'$ is an epimorphism, then M' is an injective valuation module too.*

Lemma 3.11. *Let R be a valuation domain and M a divisible torsion-free R-module. Then M is an injective valuation module.*

Proposition 3.12. *Let R be an notherian valuation ring and M a R-module divisible. Then M is an injective module.*

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